10TH INTERNATIONAL COMMAND AND CONTROL RESEARCH AND TECHNOLOGY SYMPOSIUM

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Title:
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Identification of Data Sets for a Robustness Analysis

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Abstract

In military context, the process of planning operations involves the assessment of the situation, the generation of courses of actions, and their evaluation according to significant point-of-views, in order to select the course of actions that represents the best possible compromise. Since several conflicting and quite incommensurable criteria need to be considered and balanced to make wise decisions, multicriterion decision Aid (MCDA) has been used to develop decision support systems. Given that it is impossible to derive exact models of the situation, it is required that the decision analysis procedure helps identifying "good" results despite the imperfection of the models. In particular, a robustness analysis procedure should consider all plausible values according to the model of the decision-maker's preferences in order to produce a ranking of best compromise. Defence Research and Development Canada – Valcartier (DRDC Valcartier) has developed and implemented a robustness analysis, appropriate for a multicriterion decision-support system in a context of military decision-making. The procedure proposed for the determination of a robust result contains four (4) steps. This paper presents the first step of this procedure: the establishment of data sets covering the domain of the possibilities concerning the modelling of the decision-maker's preferences.

1 Introduction

Since 1991, Defence Research and Development Canada – Valcartier (DRDC Valcartier) has been investigating different approaches to support the decision-making processes that address military command and control problems. One of these investigations was the study of a decision-aid system for the command and control of military resources within the context of Canadian airspace protection. In such a context, several conflicting and quite incommensurable criteria need to be considered and balanced to make wise decisions. As our understanding of military command and control processes improved with time, it appeared that most of the military operational planning processes were also considering several conflicting and incommensurable criteria while making a decision. Accordingly, multicriterion decision-aid process was deemed to be appropriate, in order to deal with most of military operational planning activities.

A multicriterion decision-aid process is composed of several steps. The first one is a structuration step. It aims at identifying and formalizing the basic elements of a decision-making situation under the shape of the model (Alternatives, Attributes/Criteria, Evaluations), also represented by (A, $\mathcal{A}/C$, E). The data supplied by the model (A, $\mathcal{A}/C$, E) are completed with the introduction of some elements of the decision-maker's preference modelling (M). Accordingly to the multicriterion method in use, those elements are composed of coefficients of relative importance (c.r.i.) for the attributes/criteria, and/or of parameters such as thresholds (of indifference, preference, or veto). The establishment of these values for each attribute/criterion permit to obtain the "local preferences". Afterward, these local preferences are aggregated and exploited according to the decisional problematic retained (a choice of a best alternative, a sorting of the alternatives into different categories, or a ranking of the alternatives), to obtain one or several recommendations at the end of the process.

Due to the complexity of the military operation context, it is very difficult for the military decision-makers to determine very precise values for these different coefficients and parameters while modelling preferences. The likelihood of having more than one plausible data set for these parameters leads to the possibility to get more that one result of better compromise for a single decision-making situation. Accordingly, it became a requirement for the military decision-makers to have a decision support system that provides valid results, despite the difficulty to develop an exact model of the situation. Such results, qualified as "robust", would be less influenced by the imperfection of the data occurring in the evaluations of the courses of actions as well as in the instantiation of parameters representing decision-maker's preferences during the modeling process of a military situation.

The development of a robustness analysis, appropriate for a multicriterion decision-support system in a context of military decision-making, was then conducted [1]. From this perspective, a result is considered as "robust" if it is not too far away from or not too contradictory [2] with, different results obtained for all the data sets representing plausible values for the decision-making model. This robustness concept for the military needs was inspired by the robustness definitions of Kouvelis and Yu [3]:

- The solution of a mathematical program is robust, from the point of view of the optimality, if it remains nearby the optimum whatever is the plausible set of data of the model (however, in this report it is not a question of optimality, but of better compromise).
- A specific set of data instantiates a potential realization of the model of decision represented by a set of parameters. Since the approach of robustness is crucially based on the process of generation of these data sets, it requires a good knowledge of the environment in which the decision is made.

Three critical elements were identified to compute a robust ranking [1]:

- an approach to model all the data sets that instantiate the decision-maker's preferences, which are "not so well known";

- a method to aggregate the pre-orders generated from each data set;
- a robustness criterion suited for the decision-making situation.

This work focuses on the first element: identifying an approach to model all the data sets that instantiate the decision-maker's preferences, which are "not so well known". This paper starts with a presentation of the decision-maker's preferences modelling, and describes the associated coefficients and parameters: coefficients of relative importance (c.r.i.) of the criteria, thresholds of veto, thresholds of indifference and thresholds of preferences (section 2). Then, for each one of them, an approach is proposed to identify the different values of these c.r.i. (section 3) and parameters (section 4) that should be considered to model decision-maker's preferences. Finally, in the section 5, an approach to establish the data sets from these values is presented.

2 Decision-maker's preferences modelling

Modelling the decision-maker's preferences is not an easy task because it is difficult to apprehend the subjectivity and the imprecision/ambiguity of human behaviour. The major challenge facing the implementation of any decision aid is the "*accurate*" assessment of these preferences. Siskos [4] declared that *"... the major problem for the analyst facing multicriterion phenomena raise, is the assessment of a model reflecting at best the preferences of the decision-maker."*

Many ways and theories exists to articulate and model preferences. For example, utility functions, valued functions, pairwise comparisons, tradeoffs and discrimination thresholds could be used (see [5,6,7,8,9,10,11] for more details). In this section, we limit our review to the discrimination thresholds for preferences modelling.

Roy and Bouyssou [9] maintains that when comparing two actions (such as COA a_i and COA a_k) taking into account many criteria, a decision-maker may be in one of the following situations:

i) he/she is indifferent between a_i and a_k (denoted $a_i \sim a_k$),

ii) he/she strictly prefers a_i to a_k (denoted $a_i \succ a_k$),

iii) he/she weakly prefers a_i to a_k (hesitation between indifference and strict preference: denoted $a_i \succ^f a_k$), or

iv) he/she considers that a_i is incomparable to a_k (hesitation between $a_i \succ a_k$ and $a_k \succ a_i$, or the two COAs are *a priori* matchless: denoted $a_i \mathbf{?} a_k$).

It is evident that when considering a single criterion at a time, there is no room for incomparability. In classical decision analysis, any criterion is presumed to have an absolute discrimination power. This implies that when comparing two alternatives a_i and a_k according to an absolute criterion (called also "*true-criterion"*), we get the following preference relations (Figure 1):

$$\begin{cases} a_i \succ_{\mathbf{j}} a_k \Leftrightarrow e_{ij} > e_{kj} \\ a_i \sim_{\mathbf{j}} a_k \Leftrightarrow e_{ij} = e_{kj} \end{cases}$$

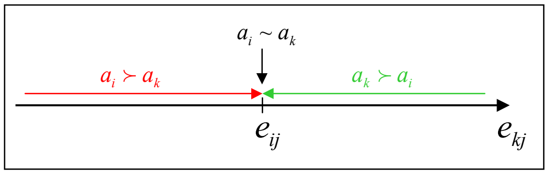

FIGURE 1 - True-criterion

In this case, the indifference is reduced to a single point where the two evaluations are equal, and otherwise we have strict preference situations. The single point indifference is called a strict indifference. However, the evaluation of a COA with regard to a criterion is often uncertain and imprecise. Moreover, on top of these uncertainties and imprecision, the decision-maker's preferences may involve some hesitations, doubts, indecision, etc. In order to take these behaviours into account, one can introduce an indifference threshold $q_j \geq 0$, such that if the performances of two alternatives on a criterion *j* (called *quasi-criterion*) differ by less than q_j, then there is an indifference relation $\sim_{\mathbf{j}}$ such as (Figure 2):

$$\begin{cases} a_i \succ_{\mathbf{j}} a_k \Leftrightarrow e_{ij} - e_{kj} \geq q_j \\ a_i \sim_{\mathbf{j}} a_k \Leftrightarrow \left| e_{ij} - e_{kj} \right| < q_j \end{cases}$$

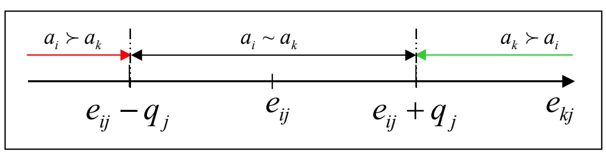

FIGURE 2 - Quasi-criterion

In this case, the hesitation range is expanded from a single point to an interval (which can be variable). If q_j is set to 0, then the *quasi-criterion* collapses to a *true criterion*. Moreover, one may define a strict preference threshold for a quasi-criterion *j*, $p_j \geq 0$, such that if the performances of a_i and a_k according to this criterion differ by at least p_j, then it is a situation where one alternative is strongly preferred $\succ_{\mathbf{j}}$ to the other one. However, if this difference is between q_j and p_j, we can conclude to a weak preference $\succ^{\mathbf{f}}_{\mathbf{j}}$ between the two COAs. A criterion with preference and indifference thresholds is a *pseudo-criterion*, which is illustrated as follows (Figure 3):

$$
\begin{cases}
a_i \succ_{\mathbf{j}} a_k \Leftrightarrow e_{ij} \geq e_{kj} + p_j \\
a_i \succ_j^f a_k \Leftrightarrow q_j < e_{ij} - e_{kj} < p_j \\
a_i \sim_{\mathbf{j}} a_k \Leftrightarrow \left| e_{ij} - e_{kj} \right| \leq q_j
\end{cases}
$$

$$
\textit{where} \quad p_j > q_j
$$

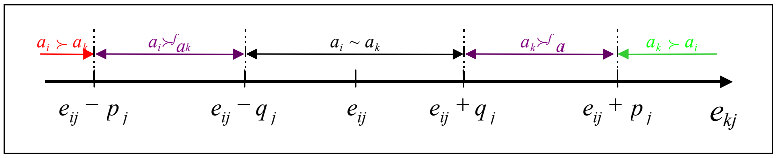

FIGURE 3 - Pseudo-criterion

If q_j is set to 0, then the *pseudo-criterion* collapses to a *pre-criterion*, and if $p_j=q_j$, then the *pseudo-criterion* collapses to a *quasi-criterion*. The use of constant thresholds for establishing preference relations is illustrated on Figure 3. However, one should note that these thresholds are not necessarily constant; i.e. $q_j = q_j(e_{ij})$ and $p_j = p_j(e_{ij})$.

An implementation of these MCDA concepts into an automated decision support system was realized and called CASAP (for "Commander's Advisory System for Airspace Protection"). The method selected for CASAP is called PAMSSEM [12,13,14] (for "*Procédure d'Agrégation Multicritère de type Surclassement de Synthèse pour Évaluations Mixtes*") and is based on the model (A, A/C, E) [14]. In a military context, there is **A** , a set of the COAs, $\mathbf{A} = \{a_1, a_2, \ldots, a_i, \ldots, a_m\}$, and **Λ/C** a coherent family of criteria, $\mathbf{\Lambda/C} = \{C_1, C_2, \ldots, C_j, \ldots, C_n\}$. The value of the i^{th} COA evaluation according to the j^{th} criterion is denoted e_{ij}. The evaluation of all the COAs according to the set of criteria produces the multiple criteria performance table **E** (see table I).

TABLE I

<u>Multiple Criteria Performance Table **E**</u>

Criteria (1…n)

CoAs (1…m)	C_1	…	C_j	…	C_n
a_1	e_{11}	…	e_{1j}	…	e_{1n}
:	:	:	:	:	:
a_i	e_{i1}	…	e_{ij}	…	e_{in}
:	:	:	:	:	:
a_m	e_{m1}	…	e_{mj}	…	e_{mn}

PAMSSEM requires a coefficient of relative importance (π_j) associated to each criterion, as well as the value of certain parameters such as discrimination thresholds (q_j representing the indifference one; p_j representing the preference one) and as veto thresholds ($v_{j,}$) to establish the local preferences:

- π_j represents the "voting power" the decision-maker is willing to assign to each criterion;
- q_j represents the highest difference between the evaluations of two alternatives according to a criterion j for which the decision-maker is incapable to make a clear choice between these two alternatives, given that everything is the same otherwise;
- p_j represents the smallest difference between the evaluations of two alternatives according to the criterion j for which the decision-maker is able to make a clear choice of one, given that everything is the same otherwise;
- ν_j represents the smallest difference between the evaluations of two alternatives according to the criterion j for which the decision-maker cannot conclude that an alternative a_i is as good as a_k, if the performance of a_k is higher than the performance of a_i and if the difference of the evaluations between them is greater than ν_j (even if the performance of a_i is higher than a_k for all others criteria).

To resume, the characteristics of our military decision-making situation context are:

- $A=(a_1,\ldots,a_i,\ldots,a_m)$;
- $\Lambda/C=(g_1,\ldots,g_j,\ldots,g_n)$;
- $E=(e_{ij}=g_j(a_i), i=1,...,m; j=1,....,n)$;
- $M=(\pi_j, v_j(e_{ij}),q_j(e_{ij}),p_j(e_{ij}), i=1,...,m; j=1,...,n)$; and
- a multicriterion method, PAMSSEM, within the framework of the ranking problematic.

given *m* alternatives and *n* attributes/criteria.

The determination of these coefficients and parameters' values are very specific to a decision-making situation, as well as the decision-maker's value system, belief and preferences. At this stage, it is requested that an analyst helps the decision-maker to determine, as much as possible, a coherent set of parameters/values that reflects the "real" decision-maker's preferences. This has to be done considering that the information is imperfect due to inaccuracies, or to the incompleteness nature of some information.

Due to the complexity of the military operation context, it is very difficult for the military decision-makers to determine very precise values for these different coefficients and parameters while modelling preferences. Then, instead of determining precise values, data sets considered as good enough to instantiate the decision-maker's preferences have to be identified.

In PAMSSEM, as it is the case in the methods of ELECTRE type, thresholds are used for the modelling of the imperfection of information related to the action evaluations and the decision-maker's preferences [15]. Since in PAMSSEM, mixed evaluations (noted E) can be admitted, the current imperfections (stochastic, fuzzy nature, missing data) related to E are thus modelled and taken into account for the determination of the multicriterion compromise. So, we agree that the data sets, for the determination of a robust result, could consist of values for coefficients of relative importance of the criteria (π_j), thresholds of veto ($v_j(e_{ij})$), thresholds of indifference ($q_j(e_{ij})$) and thresholds of preferences ($p_j(e_{ij})$). This reduces the variety and the quantity of sets of plausible values; but still this quantity can be very large.

The next section presents an approach to identify the different values of these coefficients and parameters that should be considered in the data sets that intends to model decision-maker's preferences. The data sets are determined by the available information. If the decision-maker, in interaction with the analyst, is able to supply a lot of information, all this information has to be used to determine the data sets. It is essential that all these sets cover the domain of the possibilities concerning the modelling of the decision-maker's preferences. When possible, this work considers the use of intervals to express the imprecision related to the values used to model the decision-maker's preferences, while doing a robustness analysis.

3 Coefficients of relative importance of the criteria

The coefficients of relative importance of the criteria $g_{(j)}$ have values between 0 and 1 and their sum is equal to 1:

$$0 < \pi_j < 1, \quad j = 1,\dots,n \text{ with } \sum_{j=1}^{n} \pi_j = 1 .$$

If the decision-maker can determine precise values for the importance of each criterion, these values can be used as reference to establish other data sets. If he can go as far as expressing intervals of plausible values $[\pi^1_{(j)}, \pi^2_{(j)}]$, we could then divide these intervals into small steps as wished to make a detailed analysis. In order to limit the amount of combinations that may be involved in the treatment of robustness analysis, it is suggested that only three (3) values would be used for each interval (extreme values and central one):

$$\pi^1_{(j)}, \frac{\pi^1_{(j)} + \pi^2_{(j)}}{2}, \pi^2_{(j)} \quad \forall j = 2,\dots,n-1$$

where each set of coefficients values has to be normalised.

3.1 Intervals based on decision-maker's explicit values

If the decision-maker provides explicit values for the c.r.i. of the criteria such as :

$g_{(1)},\ldots,g_{(j)},\ldots,g_{(n)}$

where $g_{(1)}$ is the most important criterion and $g_{(n)}$ the least important criterion; it is proposed to construct an interval focused on each explicit value of the coefficients. The interval for the coefficients of each criterion would be between 0 and 1 and defined as:

$$\left[\pi^1_{(1)},\pi^2_{(1)}\right],\ldots,\left[\pi^1_{(j)},\pi^2_{(j)}\right],\ldots,\left[\pi^1_{(n)},\pi^2_{(n)}\right]$$

$$\text{with}\quad \pi^1_{(n)}\rangle 0,\quad \pi^2_{(1)}\langle 1,\quad \pi^1_{(j)}\geq\pi^2_{(j+1)}\quad\text{and}\quad \pi^2_{(j)}\leq\pi^1_{(j-1)}\quad \forall j=2,\ldots,n-1$$

The limits of these intervals for the c.r.i. could be, for example in Table 1, from 10% to 20% bigger or lesser from the given value; in respecting the constraints at the limits, $\pi^1_{(j)}\geq\pi^2_{(j+1)}$.

Table 1. Intervals for the coefficients of relative importance of the criteria

g_j	π^0_j	$\pi^0_j \pm 10\%$	$\pi^0_j \pm 20\%$
$g_{(1)}$	0.30	[0.27,0.33]	[0.24,0.36]*
$g_{(2)}$	0.24	[0.216,0.264]	[0.192,0.288]*
$\overline{g_{(3)}},\overline{\overline{g_{(3)}}}$ **	0.18,0.18	[0.162,0.198], [0.162,0.198]	[0.144,0.216], [0.144,0.216]*
$g_{(4)}$	0.10	[0.09,0.11]	[0.08,0.12]

* indicates particular cases where the constraint $\pi^1_{(j)}\geq\pi^2_{(j+1)}$ is not respected

** $\overline{g_{(3)}},\overline{\overline{g_{(3)}}}$ represents two different criteria having the same value $g_{(3)}$

Considering that three (3) values are used for each interval (extreme values and central one), and that each set of coefficients has to be normalised leads to the identification of 3^n-2 sets of coefficients (-2 since the combination of extreme values $\pi^1_{(j)}$ and $\pi^2_{(j)},\quad j=1,\ldots,n$ are normalised; that leads to the same set of parameters as the normalised central values $\overline{\pi_{(j)}}=\dfrac{\pi^1_{(j)}+\pi^2_{(j)}}{2}$).

In the particular cases where the constraints are not respected (such as the ones with an * in the Table 1), we could use the center between the limits: i.e. if $\pi^1_{(j)}\geq\pi^2_{(j+1)}$ then, by considering $\overline{\pi^1_{(j)}} = \overline{\pi^2_{(j+1)}} = \dfrac{\pi^1_{(j)} + \pi^2_{(j+1)}}{2}$ we get $\overline{\pi^1_{(j)}} \le \pi^0_{(j+1)}$ and $\overline{\pi^2_{(j)}} \ge \pi^0_{(j+1)}$. For our example, since $\pi^1_{(1)} = 0.24 < \pi^2_{(2)} = 0.288$, we can use $\overline{\pi^1_{(1)}} = \overline{\pi^2_{(2)}} = \dfrac{0.24 + 0.288}{2} = 0.264$ (<0.30 and >0.24).

In this case, the combination of normalised extreme values does not lead to the same set of parameters as the combination of normalised central values, and we will get 3^n sets of parameters.

3.2 Intervals based on decision-maker's intervals

If the decision-maker is able to provide an interval for the coefficients of each criteria, such as:

$$\left[\pi^1_{(1)}, \pi^2_{(1)}\right], \ldots, \left[\pi^1_{(j)}, \pi^2_{(j)}\right], \ldots, \left[\pi^1_{(n)}, \pi^2_{(n)}\right] \text{ with } \pi^1_{(n)} \rangle 0 \text{ and } \pi^2_{(1)} \langle 1$$

and if the relative importance of the criterion $g_{(1)}$ is greater than the relative importance of criteria $g_{(2)}$ and so on, where $g_{(1)} \to \left[\pi^1_{(1)}, \pi^2_{(1)}\right]$, $g_{(j)} \to \left[\pi^1_{(j)}, \pi^2_{(j)}\right]$ and $g_{(n)} \to \left[\pi^1_{(n)}, \pi^2_{(n)}\right]$, we would use these intervals directly.

If there are no *ex æquo* criteria (i.e. there is no criterion that have the same importance as another one), it is possible to identify a limited number of values for each interval. Considering that three (3) values are used for each interval (extreme values and central one), and that each set of coefficients has to be normalised, this leads to the identification of $3^n - 2$ sets of coefficients (for the same reasons as mentioned in 3.1).

It is possible to have two (or more) criteria *ex æquo* (i.e. if two criteria have the same importance), and this may appear in more than one place in the pre-order. These *ex æquo* criteria have to be processed as one (i.e. one block). For example, if there are only two criteria that have the same relative importance, we will have, in order :
$g_{(1)}, \ldots, \left(g'_{(j)}, g''_{(j)}\right), \ldots, g_{\left(n^1 = n-1\right)}$.

The same process is done when there are two criteria *ex æquo* or if there are other groups of ex æquo criteria. For example, with eight (8) criteria respecting the following pre-order:

$$g_{(1)}, \left(\overline{g_{(2)}}, \overline{\overline{g_{(2)}}}\right), g_{(3)}, \left(\overline{g_{(4)}}, \overline{\overline{g_{(4)}}}, \overline{\overline{\overline{g_{(4)}}}}\right), g_{(5)}$$

where $g_{(1)}$ is the most important criterion and $g_{(5)}$ the least important criterion.

We have:

$$
\begin{array}{l||l}
g_{(1)} & \left[\pi^1_{(1)}, \pi^2_{(1)}\right] \\
(\overline{g_{(2)}}, \overline{\overline{g_{(2)}}}) & \left[\pi^1_{(2)}, \pi^2_{(2)}\right], \left[\pi^1_{(2)}, \pi^2_{(2)}\right] \\
g_{(3)} & \left[\pi^1_{(3)}, \pi^2_{(3)}\right] \\
\left(\overline{g_{(4)}}, \overline{\overline{g_{(4)}}}, \overline{\overline{\overline{g_{(4)}}}}\right) & \left[\pi^1_{(4)}, \pi^2_{(4)}\right], \left[\pi^1_{(4)}, \pi^2_{(4)}\right], \left[\pi^1_{(4)}, \pi^2_{(4)}\right] \\
g_{(5)} & \left[\pi^1_{(5)}, \pi^2_{(5)}\right]
\end{array}
$$

Each one of the three criteria at the fourth rank has the same interval of values for its coefficient of relative importance; then we can retain the hypothesis that the value of the importance of the coefficient for the criteria that are *ex æquo* is the same. This hypothesis leads us to keep $3^{n'} - 2$ (in our example: $3^5 - 2$) sets of coefficients, if we use three values by interval.

3.3 Intervals based on a pre-order of importance

It is possible that the decision-maker is only able to provide a pre-order on these coefficients

$$\pi_{(1)} \geq \pi_{(2)} \geq \ldots . \geq \pi_{(n)} \text{ with } \pi_j > 0 \text{ and } \sum_{j=1}^{n} \pi_j = 1,$$

and eventually the presence of equals ("*ex æquo*"). For example, in this case, we can retain six value (data) sets $\underline{\pi}^1$, $\underline{\pi}^2$, ..., $\underline{\pi}^6$ where $\underline{\pi}^1$ corresponds to the case where the criteria are equally balanced and $\underline{\pi}^6$ corresponds to the case where the c.r.i. are decreasing from 1 to 1/n (before the normalisation of the coefficients of relative importance of this vector). For the four data sets between $\underline{\pi}^1$ and $\underline{\pi}^6$, we progressively reduce the values of the c.r.i. by slices, on a basis of *(n-2)/4*, in respecting the pre-order. If *(n-2)/4* is not an integer, then the fractional part has to be considered. For example, if we have 15 criteria to consider, then (n-2)/4 = 3.25 and the four (4) less important criteria are going to be modified, i.e. $\underline{\pi}^2$: (*n-1/n, n-2/n, n-3/n, n-4/n*) and progressively by slices of three criteria for the three other data sets $\underline{\pi}^3$, $\underline{\pi}^4$, $\underline{\pi}^5$. These data sets are illustrated in the Table 2.

Table 2. *Six data sets*

π^1	π^2	π^3	π^4	π^5	π^6
1	1	1	1	1	1
1	1	1	1	1	$\frac{n-1}{n}$
1	1	1	1	$\frac{n-1}{n}$	$\frac{n-2}{n}$
1	1	1	1	$\frac{n-2}{n}$	$\frac{n-3}{n}$
1	1	1	1	$\frac{n-3}{n}$	$\frac{n-4}{n}$
1	1	1	$\frac{n-1}{n}$	$\frac{n-4}{n}$	$\frac{n-5}{n}$
1	1	1	$\frac{n-2}{n}$	$\frac{n-5}{n}$	$\frac{n-6}{n}$
1	1	1	$\frac{n-3}{n}$	$\frac{n-6}{n}$	$\frac{n-7}{n}$
1	1	$\frac{n-1}{n}$	$\frac{n-4}{n}$	$\frac{n-7}{n}$	$\frac{n-8}{n}$
1	1	$\frac{n-2}{n}$	$\frac{n-5}{n}$	$\frac{n-8}{n}$	$\frac{n-9}{n}$
1	1	$\frac{n-3}{n}$	$\frac{n-6}{n}$	$\frac{n-9}{n}$	$\frac{n-10}{n}$
1	$\frac{n-1}{n}$	$\frac{n-4}{n}$	$\frac{n-7}{n}$	$\frac{n-10}{n}$	$\frac{n-11}{n}$
1	$\frac{n-2}{n}$	$\frac{n-5}{n}$	$\frac{n-8}{n}$	$\frac{n-11}{n}$	$\frac{n-12}{n}$
1	$\frac{n-3}{n}$	$\frac{n-6}{n}$	$\frac{n-9}{n}$	$\frac{n-12}{n}$	$\frac{n-13}{n}$
1	$\frac{n-4}{n}$	$\frac{n-7}{n}$	$\frac{n-10}{n}$	$\frac{n-13}{n}$	$\frac{1}{n}$

4 Threshold values

Our work considers the use of intervals to express the imprecision related to the values that are used for modelling the decision-maker preferences, while doing a robustness analysis. The three types of thresholds that are part of the decision-maker preferences' model in PAMSSEM are: indifference threshold, preference threshold and veto threshold. Our work is looking at criteria that have cardinal evaluations.

4.1 Indifference threshold

We need to get an interval for the indifference threshold such as :

$$\left[q_j^1, q_j^2\right] \quad \forall j, \quad \text{with} \quad q_j^1 \geq 0 .$$

The intervals of the indifference threshold (q_j) can vary between 0 and E_j, which corresponds to the range of the scale associated to the criteria g_j.

If the decision-maker provides an interval for this indifference threshold, then we are going to use it directly.

If the decision-maker provides an exact value for this threshold, we propose to define the interval by using two other values: 80% and 60% of $q_j^{'}$, for example $q_j^2 = 0.8q_j^1$ and $q_j^3 = 0.6q_j^1$ are two inferior values since the reference value is probably over estimated. These values (80% and 60%) have been obtained from simulations, and therefore, they are not absolute values.

If the decision-maker is not able to provide anything for this indifference threshold, we [**14**] suggest to calculate a default value by using :

$$q_j^{'} = 0.15X0.25E_j$$

and then, one can define an interval by using 80% and 60% of these $q_j^{'}$.

4.2 Preference threshold

One needs to get an interval for the preference threshold such as :

$$\left[p_j^1, p_j^2\right] \quad \forall j \text{ with } p_j^1 \geq q_j^2 \quad \text{and} \quad p_j^2 \leq E_j .$$

The intervals of the preference threshold (p_j) can vary between q_j and E_j which corresponds to the range of the scale that is associated to the criteria g_j.

If the decision-maker provides an interval for this threshold, then we are going to use it directly.

If the decision-maker provides an exact value for this preference threshold, we propose to define the interval by using two other values: 80% and 60% of $p_j^{'}$; for example $p_j^2 = 0.8p_j^1$ and $p_j^3 = 0.6p_j^1$ are two inferior values since the reference value is probably over estimated with the constraint: $p_j^1 \geq q_j^2 \quad \forall j$. These values (80% and 60%) have been obtained from simulations, and therefore, they are not absolute values.

If the decision-maker is not able to provide anything for this preference threshold, we [**14**] suggest to calculate a default value by using :

$$p_j^{'} = 0.25E_j + 0.05(v_j - q_j),$$

where v_j is the veto threshold described in subsection 4.3

and then to define an interval by using 80% and 60% of $p_j^{'}$.

4.3 Veto threshold

If the decision-maker provides an interval for the veto threshold, such as

$$v_j \Rightarrow \left[v_j^1, v_j^2\right]$$

then we are going to use it directly.

If the decision-maker provides an exact value for this threshold, we propose to determine the interval:

$$v_j \Rightarrow \left[v_j^1, v_j^2\right] \quad \text{with} \quad v_j^1 > p_j^2$$

by using two other values: 80% and 60% of $v_j^{'}$. For example, $v_j^2 = 0.8v_j^1$ and $v_j^3 = 0.6v_j^1$ are two inferior values since the reference value is probably over estimated with the constraint :

$$v_j \geq p_j \quad \forall j$$

These values (80% and 60%) have been obtained from simulations, and therefore, they are not absolute values.

If the decision-maker is not able to provide any exact value for this threshold, we [**14**] suggest to calculate a default value such as:

$$v_j = \frac{0.25E_j}{\sqrt{\pi_j}}$$ where E_j is the range of the scale associated with the criterion g_j and π_j his c.r.i..

However, we have to make sure that $v_j > p_j^2 \quad \forall j$.

Since this value varies in function of the coefficients of relative importance of the criteria, no interval will be defined.

5 Identification of plausible data sets

Considering all plausible parameter values will determine the number of data sets to handle in the robustness analysis. As mentioned previously, this work considers the use of intervals to express the imprecision related to the values that are used for modelling the decision-maker's preferences while doing a robustness analysis. Accordingly, the number of data sets to be handled in a robustness analysis can be rather big. Even by limiting the number of values to three (3) by each interval, the number of data sets can be quite impressive. Even if this is nothing unexpected (for example, Roy and Bouyssou [**16**] bring in 136 data sets in their analysis of robustness), we need to define a way to handle these combinations in an acceptable time frame. The approach we propose is to treat the different parameters as groups or blocks.

For this example, we will consider both the indifference and preference thresholds only. If there is *m* cardinal criteria, we can form 9^m sets of thresholds. We can reduce this number to 9 by grouping (block) the extreme and central values of all the criteria. For example, see Table 3:

Table 3. Treatment of parameters in blocks

	q_j			p_j		
	q_j^1	$\overline{q_j}$	q_j^2	p_j^1	$\overline{p_j}$	p_j^2
g_1	q_1^1	$\overline{q_1}$	q_1^2	p_1^1	$\overline{p_1}$	p_1^2
…	…	…	…	…	…	…
g_j	q_j^1	$\overline{q_j}$	q_j^2	p_j^1	$\overline{p_j}$	p_j^2
…	…	…	…	…	…	…
g_m	q_m^1	$\overline{q_m}$	q_m^2	p_m^1	$\overline{p_m}$	p_m^2

By computing the values in blocks, we will get 9 combinations instead of having 9^3 ones. Therefore, we suggest that such an approach should be used to reduce the number of combinations in order to obtain an acceptable time frame for execution of the robustness analysis.

Formally, we designate by $J_t=(\pi_t, v_t, q_t, p_t)$, a selected plausible data set for the development of a robust result. It is necessary to note that J_t is not a data of the problem, but the result of an interactive process with the decision-maker [**17**]. Our robustness analysis plan will consider s data sets ($J_1,\ldots,J_s;\ t=1,\ldots,s$) forming a representative subset of all the sets of plausible values.

6 Conclusion

According to PAMSSEM, the multicriterion method used, the elements for the decision-maker's preferences modelling are composed of coefficients of relative importance for the attributes/criteria and thresholds (of indifference, preference and veto). Due to the complexity of the military operation context, it is very difficult for the military decision-makers to determine very precise values for the different coefficients and parameters while modelling preferences. The likelihood of having more than one plausible data set for these parameters leads to the possibility to get more that one result of better compromise for a single decision-making situation study. Accordingly, it became a requirement for the military decision-makers to have a decision support system that provides valid results, despite the difficulty to develop an exact model of the situation. Such results, qualified as "robust", would be less influenced by the imperfection of the data occurring in the instantiation of parameters representing decision-maker's preferences during the modeling process of a military situation. Thus, a result will be considered as "robust" if it is not too far away from or not too contradictory [2] with, different results obtained for all the data sets representing plausible values for the decision-making model.

This work describes an approach to model all the data sets that instantiate the decision-maker's preferences, which are "not so well known". It introduces the decision-maker's preferences modelling, and describes the associated coefficients and parameters: coefficients of relative importance of the criteria (π_j), thresholds of veto ($v_j(e_{ij})$), thresholds of indifference ($q_j(e_{ij})$) and thresholds of preferences ($p_j(e_{ij})$). For each one of them, an approach is proposed to identify the different values that should be considered in the data sets to model decision-maker's preferences. An approach for the establishment of these data sets is also proposed. The data sets are determined by the available information. If the decision-maker, in interaction with the analyst, is able to supply a lot of information, all this information has to be used to determine the data sets. It is essential that all these sets cover the domain of the possibilities concerning the modelling of the decision-maker's preferences. When possible, this work considers the use of intervals to express the imprecision related to the values put in the model of the decision-maker's preferences, while doing a robustness analysis.

Besides, one can wonder if it is sufficient to consider only the imperfection of information at the modelling level of the decision-maker's preference. Even if the procedure of multicriterion aggregation used in CASAP prototype treats distributional evaluations, it is possible that the distributional evaluation of an alternative according to a criterion would not be unique. As proposed by Roy [18], the fuzzy subsets language could be used to represent the values badly known by the parameters that are required to model preferences.

7 References

1. Martel, J.-M., Bélanger, M., Guitouni, A. (2005). Robustness Analysis for a Multicriterion Decision-Aid Process. DRDC TR 2004-379. Defence Research and Development – Valcartier.
2. Vincke, Ph. (1999). Robust Solutions and Methods in Decision-Aid. *Journal of Multi-Criteria Decision Analysis*, 8 (3), 181-187.
3. Kouvelis, P., and Yu, G. (1997). Robust Discrete Optimization and Its Applications. Kluver Academic Publishers, Netherlands.
4. Siskos, J. (1982). A Way to Deal with Fuzzy Preferences in Multi-Criteria Decision Problems, *European Journal of Operational Research*, No 10, pp. 314-324.
5. Fishburn, P. C. (1970). Utility Theory for Decision Making, John Wiley and Sons, New York.
6. Keeney R. and Raifaa, H. (1976). Decision with Multiple Objectives; Preferences and Value Trade-Offs, Wiley, New York.
7. Keeney, R. (1992). Value Focused Thinking: A Path to creative Decision Making, Cambridge: Harvard University Press.

8. Roubens, M. and Vincke, Ph. (1985). Preference Modelling, Springer-Verlag, Berlin.

9. Roy, B. et Bouyssou, D. (1993). Aide Multicritère à la décision: Méthodes et Cas, Economica, Paris.

10. Saaty, T. L. (1980). The Analytic Hierarchy Process, McGraw-Hill, New York.

11. Vincke, Ph. (1989). L'aide multicritère à la décision, Éditions de l'Université de Bruxelles, Brussels.

12. Martel, J. M., Kiss, L. R., and Rousseau, A. (1997). PAMSSEM: Procédure d'Agrégation Multicritère de type Surclassement de Synthèse pour Évaluation Mixtes. Faculté des sciences de l'Administration, Université Laval.

13. Guitouni, A., Martel, J. M. , Bélanger, M., and Hunter, C. (1999). Managing a Decision-Making Situation in the Context of Canadian Airspace Protection (CAP), (Document de Travail 1999-021), Faculté des sciences de l'Administration, Université Laval.

14. Guitouni, A., Bélanger, M., and Martel, J. M. (2001). A Multiple Criteria Aggregation Procedure for the Evaluation of Courses of Action in the Context of the Canadian Airspace Protection. (Technical Report DREV TR 1999-215). Defence Research Establishment Valcartier.

15. Rogers, M., and Bruen, M. (1998). Choosing Realistic values of Indifference, Preference and Veto threshold for use with Environnemental Criteria within ELECTRE, *EJOR*, 107, 542-551.

16. Roy, B., and Bouyssou, D. (1986). Comparaison of two decision Aid Models Applied to a Nuclear Plant Sitting Example. *EJOR*, 25, 200-215.

17. Roy, B. (1989). Main Sources of Inaccurate Determination, Uncertainty and Imprecision in Decision Models. *Mathematical Computing Modeling*, 12 (10/11), 1245-1254.

18. Roy, B. (1997). Une lacune en RO-AD: les conclusions robustes. (Cahier du LAMSADE no144), Université Paris-Dauphine, Paris.