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Identification of Data Sets for a Robustness Analysis

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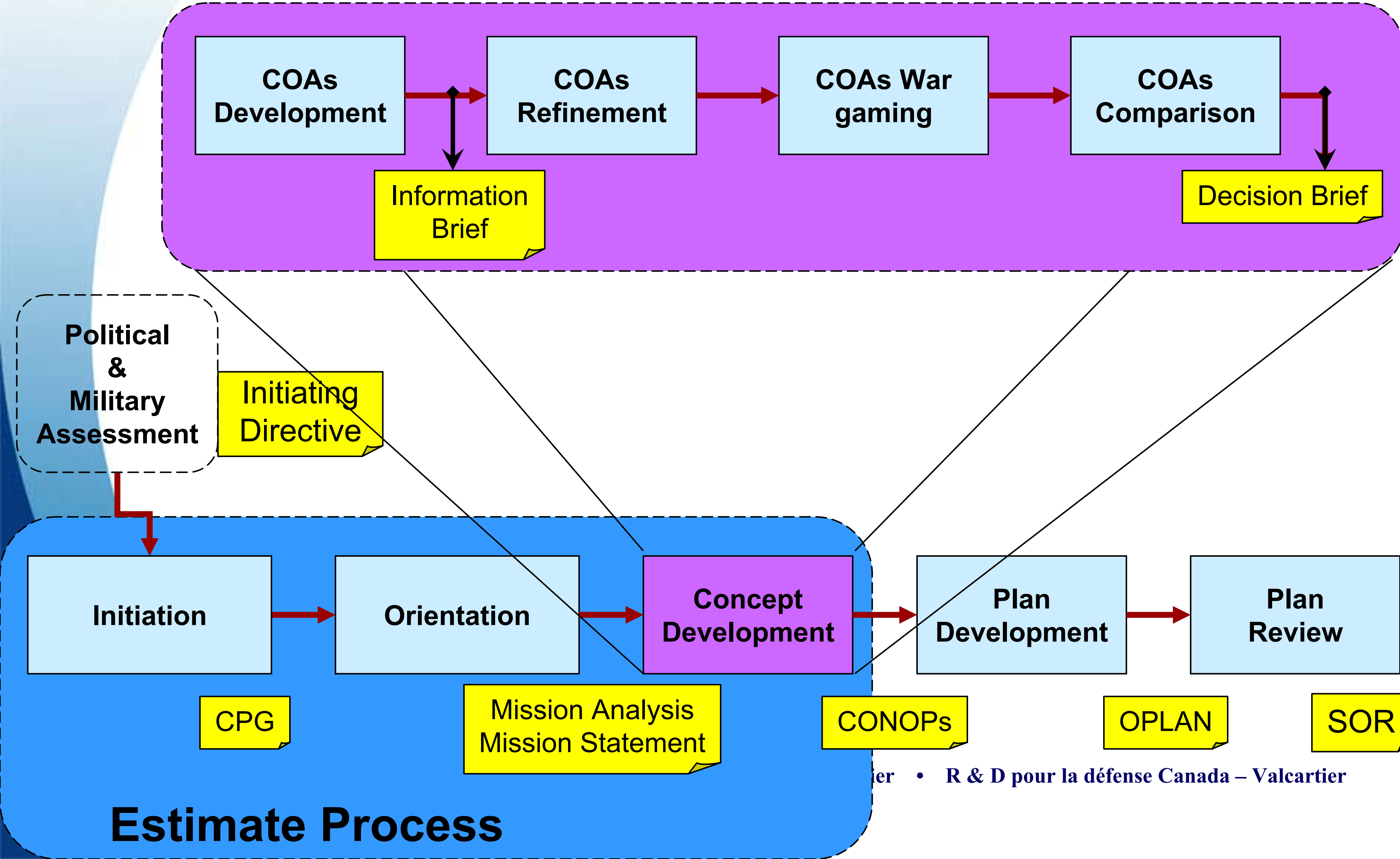
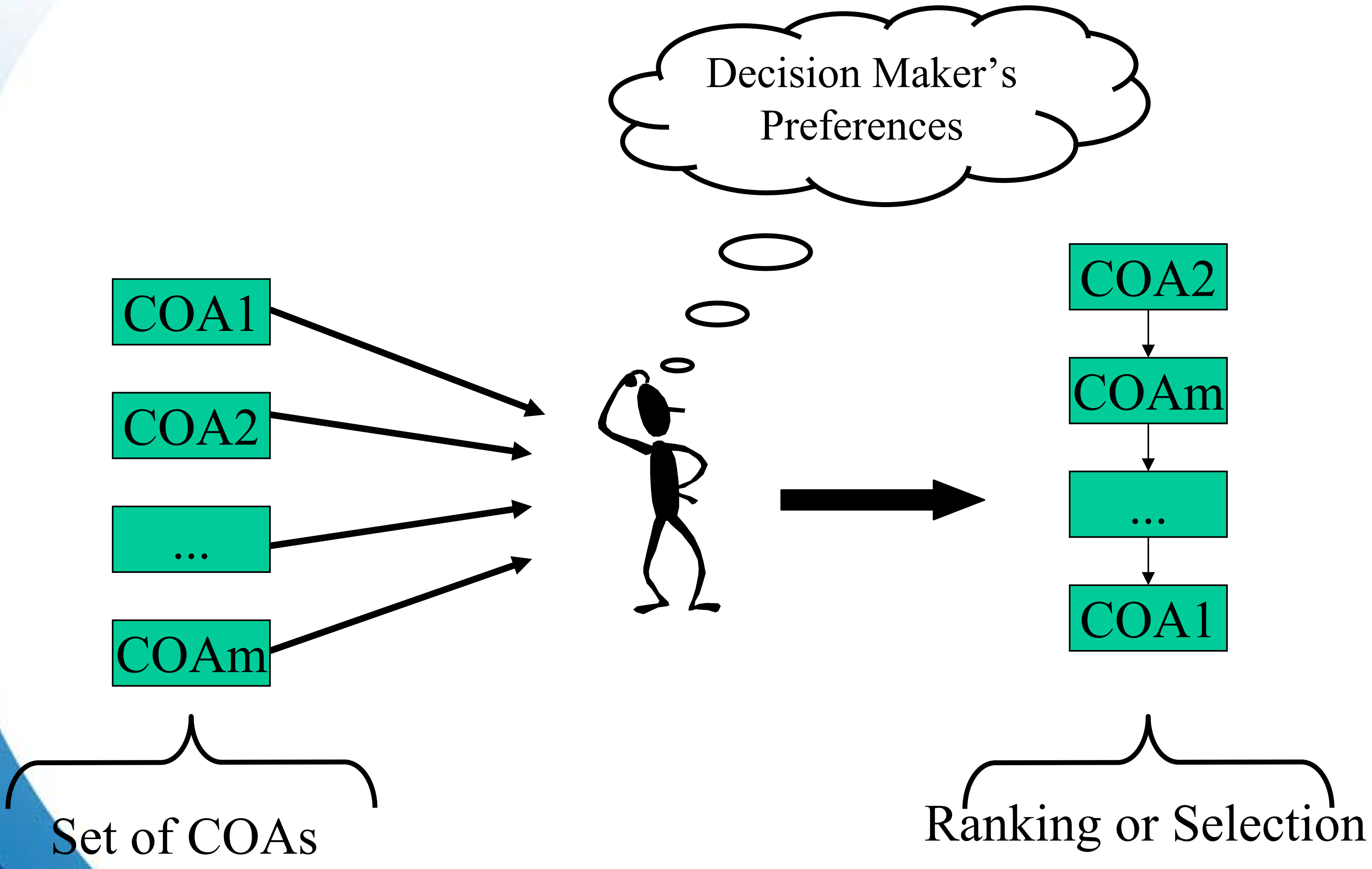
Agenda

- Canadian Forces Operations Planning Process (CFOPP)
- Multicriteria Decision Aid (MCDA) Methodology
- Robustness Analysis
- Identification of possible data sets based on DM local preferences
- Identification of plausible data sets

Canadian OPP and Estimate Process



Comparison of COAs



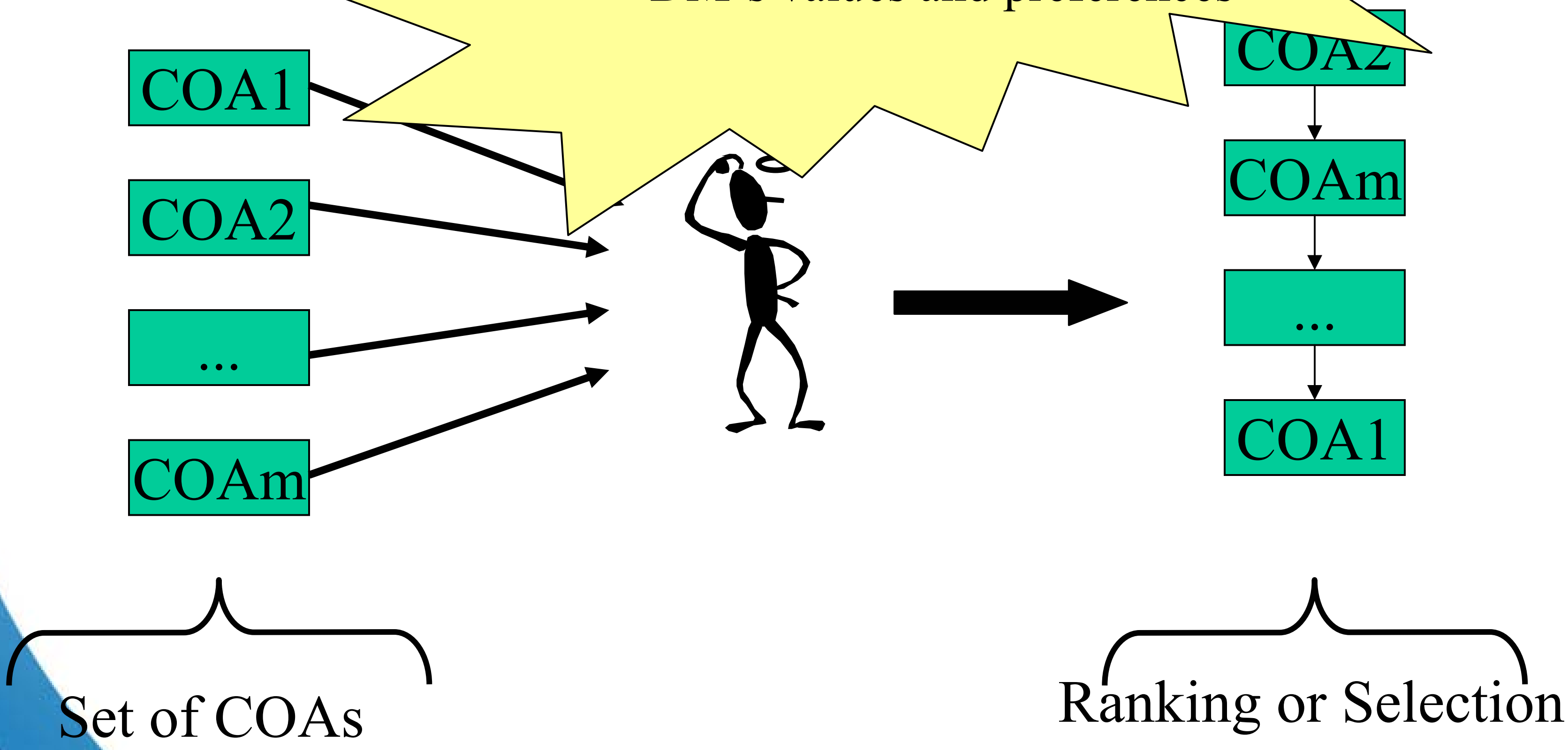
Initial Decision Maker's Preference: COAs Evaluation Criteria

Factors	Criteria
Flexibility	Covering Operational Tasks
	Covering Enemy's CoAs
	Covering Mission's Locations
Risk	Personnel Effectiveness
	Impact of Sensors Coverage Gaps
	Military Personnel Loss
	Collateral Damages
	Confrontation Risk
	Equipment Reliability
Complexity	Operation Complexity
	Logistic Complexity
	C&C Complexity
Sustainability	Sustainability
Optimum Use of Resources	Cost of Resources

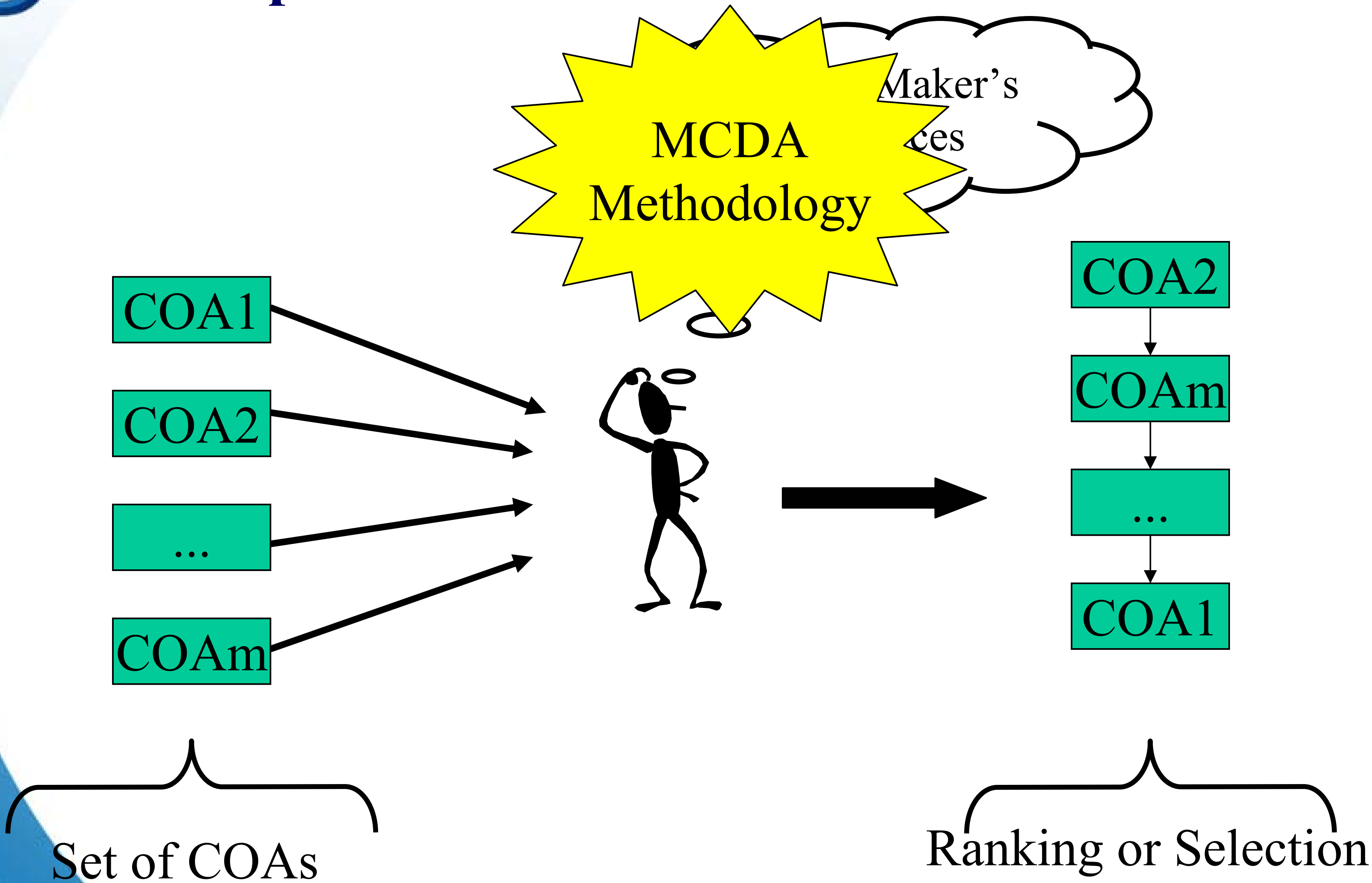
Comparison of COAs

Best possible compromise considering:

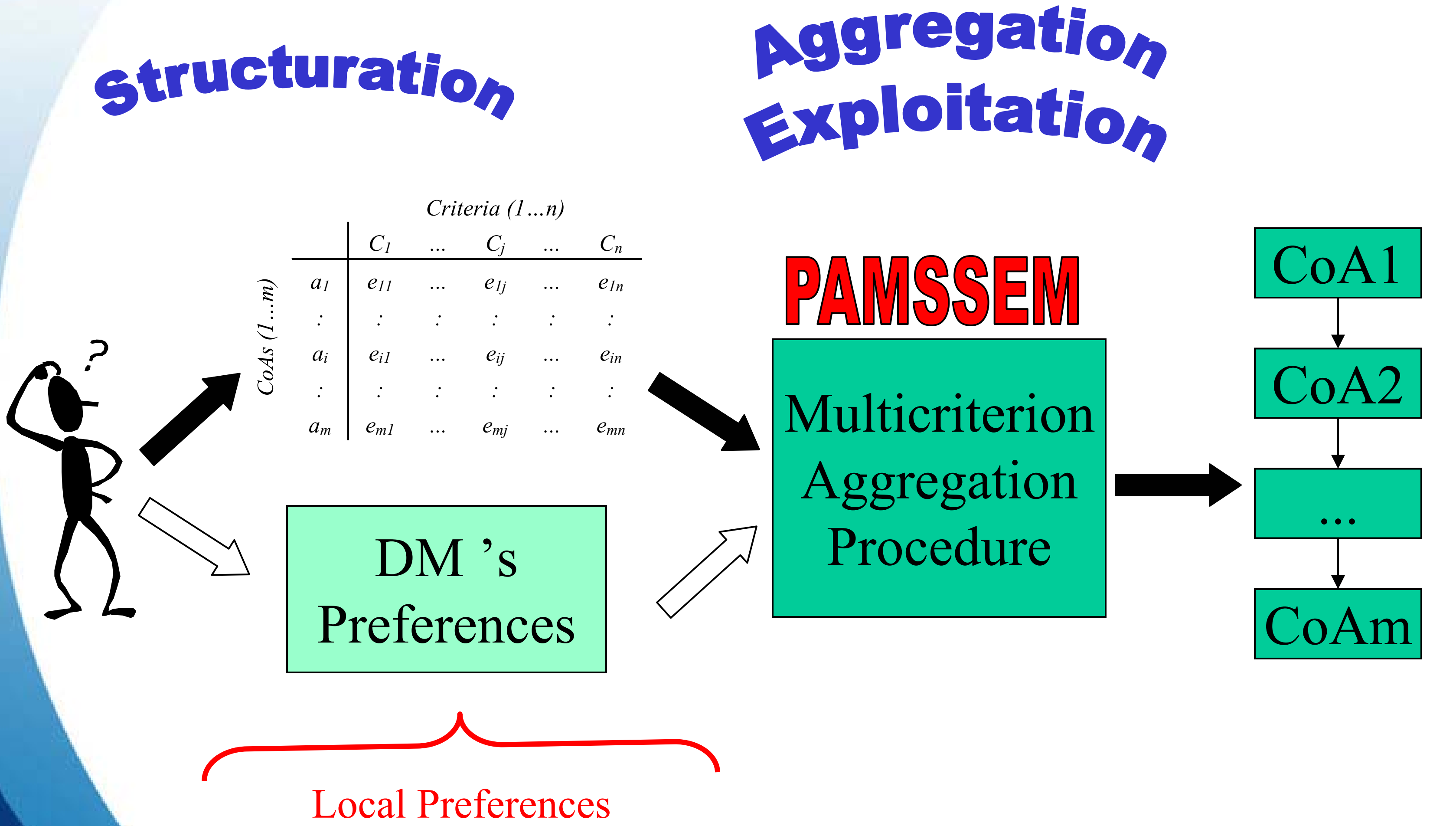
- Conflicting evaluation criteria
- DM's values and preferences



Comparison of COAs



Applying MCDA Methodology to Compare COAs



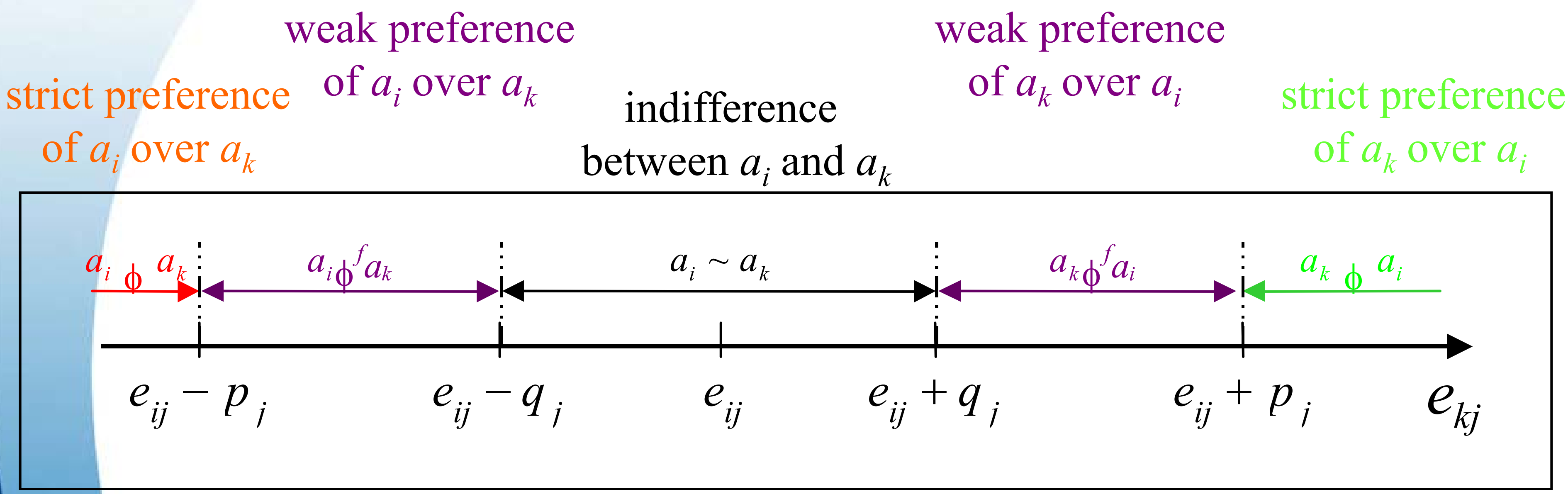
Modelling Decision Making Styles: Introduction of Local Preferences Modelling

- Each criterion is assigned a coefficient of relative importance (π_j), which might represents
  - a “trade-off” or a “voting power”
- When comparing two COAs, three types of thresholds are introduced
  - Indifference (q_j) thresholds
  - Preference (p_j) thresholds
  - Veto (v_j) thresholds

Modelling Decision Making Styles: Introduction of Local Preferences Modelling (2)

- Indifference (q_j) thresholds represent:
  - the highest difference between the evaluations of two COAs, according to a given criterion *j,* for which the decision-maker is incapable to make a clear choice between these two alternatives, given that everything is the same otherwise
- Preference (p_j) thresholds represent:
  - the smallest difference between the evaluations of two alternatives, according to a given criterion *j*, for which the decision-maker is able to make a clear choice of one, given that everything is the same otherwise
- Veto (v_j) thresholds represent:
  - the smallest difference between the evaluations of two alternatives, according to a given criterion *j*, for which the decision-maker cannot conclude that an alternative a_i is as good as a_k, if the performance of a_k is higher than the performance of a_i and if the difference of the evaluations between them is greater than v_j (even if the performance of a_i is higher than a_k for all others criteria).

Possible Preference Relationship When Comparing two COAs



Discrimination Thresholds

$$
\begin{cases}
a_i \ \phi_{\mathbf{j}} \ a_k \Leftrightarrow e_{ij} \geq e_{kj} + p_j \\
a_i \ \phi_j^f \ a_k \Leftrightarrow q_j < e_{ij} - e_{kj} < p_j \\
a_i \sim_{\mathbf{j}} a_k \Leftrightarrow \left| e_{ij} - e_{kj} \right| \leq q_j
\end{cases}
$$

$$\textit{where} \quad p_j > q_j$$

Problematic

- Modelling requires transforming, reduction and decomposition of the reality
  - It is impossible to derive exact models of the situation
- The complexity of the military operation context prevents from deriving exact and precise values to represent Commanders preferences structures (command style)
- Very high likelihood for more than one plausible data set to represent the Decision Maker's preferences structure
  - Possibility to get more that one "optimal" solution for the same decision-making situation

Why a Robustness Analysis

- The imperfection of the data set obtained should be properly considered in decision analysis
- Robustness analysis should consider all plausible data sets in order to identify a robust ranking of plausible good decisions (COAs)
- A specific set of data instantiates a potential realization of the model of decision.

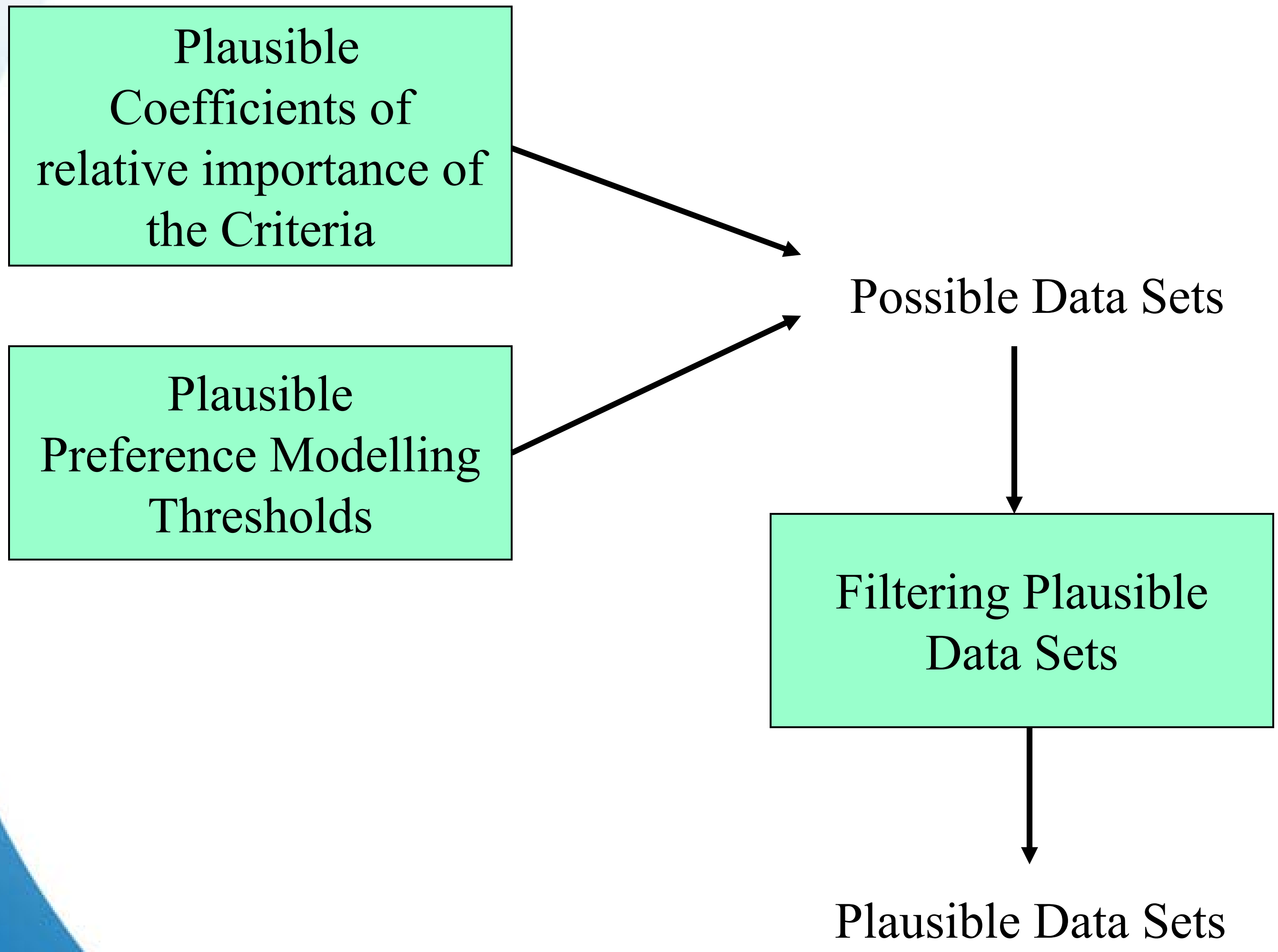
Robustness [Kouvelis and Yu, 1997]

- From the point of view of the optimality
  - The solution of a mathematical program is qualified as robust, if it remains in neighbourhood of the optimum for all plausible data sets of the model
- Generalisation from optimality to best compromise.
- Since the approach of robustness is crucially based on the process of generation of plausible data sets, it requires a good knowledge of the environment in which the decision take place

Robust ranking approach proposed

- Three critical steps were identified for robustness analysis in the context of COAs comparison:
  - an approach to model all the data sets that instantiate the decision-maker's preferences, which are "not so well known";
  - a method to aggregate the pre-orders generated from each data set;
  - a robustness criterion suited for the decision-making situation.

Identification of Possible Data Sets



Coefficients of relative importance of the criteria (CRIC)

- Identification of intervals

$$[\pi^1_{(j)}, \pi^2_{(j)}] \qquad 0 < \pi_j < 1, \quad j = 1, \ldots, n$$

  - based on decision-maker's intervals
  - based on decision-maker's explicit values

CRIC Based on decision-maker's explicit values

$$\left[\pi^1_{(1)}, \pi^2_{(1)}\right], \ldots, \left[\pi^1_{(j)}, \pi^2_{(j)}\right], \ldots, \left[\pi^1_{(n)}, \pi^2_{(n)}\right]$$

with $\pi^1_{(n)} > 0, \quad \pi^2_{(1)} < 1, \quad \pi^1_{(j)} \geq \pi^2_{(j+1)}$ and $\pi^2_{(j)} \leq \pi^1_{(j-1)} \quad \forall j = 2, \ldots, n-1$

$g_{(1)}, \ldots, g_{(j)}, \ldots, g_{(n)}$
- $g_{(n)}$ is the least important criterion
- $g_{(1)}$ is the most important criterion

g_j	π^0_j	$\pi^0_j \pm 10\%$	$\pi^0_j \pm 20\%$
$g_{(1)}$	0.30	[0.27,0.33]	[0.24,0.36]*
$g_{(2)}$	0.24	[0.216,0.264]	[0.192,0.288]*
$\overline{g_{(3)}}, \overline{\overline{g_{(3)}}}$ **	0.18,0.18	[0.162,0.198], [0.162,0.198]	[0.144,0.216], [0.144,0.216]*
$g_{(4)}$	0.10	[0.09,0.11]	[0.08,0.12]

* indicates particular cases where the constraint $\pi^1_{(j)} \geq \pi^2_{(j+1)}$ is not respected

** $\overline{g_{(3)}}, \overline{\overline{g_{(3)}}}$ represents two different criteria having the same value $g_{(3)}$

CRIC Based on decision-maker's explicit values (2)

Normalized with: $$\overline{\pi_{(j)}} = \frac{\pi^1_{(j)} + \pi^2_{(j)}}{2}$$

g_j	π^0_j	$\pi^0_j \pm 10\%$	$\pi^0_j \pm 20\%$	Normalized
$g_{(1)}$	0.30	[0.27,0.33]	[0.24,0.36]*	[0.264,0.36]
$g_{(2)}$	0.24	[0.216,0.264]	[0.192,0.288]*	[0.204,0.264]
$\overline{g_{(3)}}, \overline{\overline{g_{(3)}}}$ **	0.18,0.18	[0.162,0.198], [0.162,0.198]	[0.144,0.216], [0.144,0.216]*	[0.144,0.204], [0.144,0.204]
$g_{(4)}$	0.10	[0.09,0.11]	[0.08,0.12]	[0.08,0.12]

* indicates particular cases where the constraint $\pi^1_{(j)} \ge \pi^2_{(j+1)}$ is not respected

** $\overline{g_{(3)}}, \overline{\overline{g_{(3)}}}$ represents two different criteria having the same value $g_{(3)}$

Ex oequo Intervals

Process ex oequo as one (block)

$$
\begin{array}{l|l}
g_{(1)} & \left[\pi^{1}_{(1)}, \pi^{2}_{(1)}\right] \\
\left(\overline{g_{(2)}}, \overline{\overline{g_{(2)}}}\right) & \left[\pi^{1}_{(2)}, \pi^{2}_{(2)}\right], \left[\pi^{1}_{(2)}, \pi^{2}_{(2)}\right] \\
g_{(3)} & \left[\pi^{1}_{(3)}, \pi^{2}_{(3)}\right] \\
\left(\overline{g_{(4)}}, \overline{\overline{g_{(4)}}}, \overline{\overline{\overline{g_{(4)}}}}\right) & \left[\pi^{1}_{(4)}, \pi^{2}_{(4)}\right], \left[\pi^{1}_{(4)}, \pi^{2}_{(4)}\right], \left[\pi^{1}_{(4)}, \pi^{2}_{(4)}\right] \\
g_{(5)} & \left[\pi^{1}_{(5)}, \pi^{2}_{(5)}\right]
\end{array}
$$

$$
g_{(1)}, \ldots, g_{(j)}, \ldots, g_{(n)} \quad
\begin{cases}
g_{(n)} & \text{is the least important criterion} \\
g_{(1)} & \text{is the most important criterion}
\end{cases}
$$

$$
\left[\pi^{1}_{(1)}, \pi^{2}_{(1)}\right], \ldots, \left[\pi^{1}_{(j)}, \pi^{2}_{(j)}\right], \ldots, \left[\pi^{1}_{(n)}, \pi^{2}_{(n)}\right] \quad \text{with} \quad \pi^{1}_{(n)} > 0 \quad \text{and} \quad \pi^{2}_{(1)} < 1
$$

CRIC Based on a Pre-Order of Importance

$$\pi_{(1)} \geq \pi_{(2)} \geq \ldots \geq \pi_{(n)} \text{ with } \pi_j > 0 \text{ and }, \quad \sum_{j=1}^{n} \pi_j = 1$$

Consider to have 6 data sets

- $\underline{\pi}^1 \rightarrow$ the c.r.i. are equally balanced
- $\underline{\pi}^6 \rightarrow$ the c.r.i. are decreasing from 1 to 1/n

Reduce the values by slices on a basis of

(n-2)/4

Example with 15 criteria :

$\underline{\pi}^1$	$\underline{\pi}^2$	$\underline{\pi}^3$	$\underline{\pi}^4$	$\underline{\pi}^5$	$\underline{\pi}^6$
1	1	1	1	1	1
1	1	1	1	1	$\frac{n-1}{n}$
1	1	1	1	$\frac{n-1}{n}$	$\frac{n-2}{n}$
1	1	1	1	$\frac{n-2}{n}$	$\frac{n-3}{n}$
1	1	1	1	$\frac{n-3}{n}$	$\frac{n-4}{n}$
1	1	1	$\frac{n-1}{n}$	$\frac{n-4}{n}$	$\frac{n-5}{n}$
1	1	1	$\frac{n-2}{n}$	$\frac{n-5}{n}$	$\frac{n-6}{n}$
1	1	1	$\frac{n-3}{n}$	$\frac{n-6}{n}$	$\frac{n-7}{n}$
1	1	$\frac{n-1}{n}$	$\frac{n-4}{n}$	$\frac{n-7}{n}$	$\frac{n-8}{n}$
1	1	$\frac{n-2}{n}$	$\frac{n-5}{n}$	$\frac{n-8}{n}$	$\frac{n-9}{n}$
1	1	$\frac{n-3}{n}$	$\frac{n-6}{n}$	$\frac{n-9}{n}$	$\frac{n-10}{n}$
1	$\frac{n-1}{n}$	$\frac{n-4}{n}$	$\frac{n-7}{n}$	$\frac{n-10}{n}$	$\frac{n-11}{n}$
1	$\frac{n-2}{n}$	$\frac{n-5}{n}$	$\frac{n-8}{n}$	$\frac{n-11}{n}$	$\frac{n-12}{n}$
1	$\frac{n-3}{n}$	$\frac{n-6}{n}$	$\frac{n-9}{n}$	$\frac{n-12}{n}$	$\frac{n-13}{n}$
1	$\frac{n-4}{n}$	$\frac{n-7}{n}$	$\frac{n-10}{n}$	$\frac{n-13}{n}$	$\frac{1}{n}$

Indifference Thresholds

$$\left[q_j^1, q_j^2\right] \quad \forall j, \quad \text{with} \quad q_j^1 \geq 0$$

- DM interval
- Otherwise
  - DM value

$$q_j^{'}$$

  - Default value

$$q_j^{'} = 0.15 X 0.25 E_j$$

$\rightarrow$ 80% and 60%

Preference Thresholds

$$\left[p_j^1, p_j^2\right] \quad \forall j \qquad p_j^1 \geq q_j^2 \quad \text{and} \quad p_j^2 \leq E_j$$

- DM interval
- Otherwise
  - DM value

$$p_j^{'}$$

  - Default value

$$p_j^{'} = 0.25E_j + 0.05\left(v_j - q_j\right)$$

→ 80% and 60%

Veto Thresholds

$$v_j \Rightarrow \left[v_j^1, v_j^2\right] \quad \text{with} \quad v_j^1 > p_j^2$$

- DM interval
- Otherwise
  - DM value

$$v_j$$

  - Default value

$$v_j = \frac{0.25E_j}{\sqrt{\pi_j}}$$

→ 80% and 60%

Filtering Plausible Data Sets

- To reduce the number of data sets
  - For each interval, use only 3 data
    - First value, middle one, last one
  - Treatment of parameters as groups or blocks

	q_j			p_j		
	q_j^1	$\overline{q}_j$	q_j^2	p_j^1	$\overline{p}_j$	p_j^2
g_1	q_1^1	$\overline{q}_1$	q_1^2	p_1^1	$\overline{p}_1$	p_1^2
...	...	...	...	...	...	...
g_j	q_j^1	$\overline{q}_j$	q_j^2	p_j^1	$\overline{p}_j$	p_j^2
...	...	...	...	...	...	...
g_m	q_m^1	$\overline{q}_m$	q_m^2	p_m^1	$\overline{p}_m$	p_m^2

Discussion

- Characteristics of the proposed military decision-making model
  - $A=(a_1,\ldots,a_i,\ldots,a_m)$;
  - $\Lambda/C=(g_1,\ldots,g_j,\ldots,g_n)$;
  - $E=(e_{ij} = g_j(a_i)$, $i=1,\ldots,m$; $j=1,\ldots,n)$;
  - $M=(\pi_j, v_j(e_{ij}), q_j(e_{ij}), p_j(e_{ij})$, $i=1,\ldots,m$; $j=1,\ldots,n)$; and
  - a multicriterion method, PAMSSEM, within the framework of the ranking problematic.

given *m* alternatives and *n* attributes/criteria

Discussion

- Identification of possible values for:
  - coefficient of relative importance (π_j)
  - discrimination thresholds
    - indifference (q_j)
    - preference (p_j)
  - veto thresholds (v_j)
- Identification of plausible data sets

Conclusion

- Robust results should be less influenced by the imperfection of the data occurring in the evaluations of the courses of actions as well as in the instantiation of parameters representing decision-maker's preferences during the modeling process of a military situation
- Considering all plausible information that might represent the decision-making context
  - Not constrained to a single data set
- Robustness concept should be generalised to other information/knowledge analysis methodologies
  - e.g.; IPB and Enemy's estimates

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