14TH ICCRTS

1. Title: **On Extending Temporal Models in Timed Influence Networks**

2. Topic: **Track 5: Experimentation and Analysis**

3. Authors: **Abbas K. Zaidi, Alexander H. Levis and P. Papantoni-Kazakos**

4. Point of Contact: **Abbas K. Zaidi**

5. Organization: **System Architectures Lab, George Mason University**

6. Address: **System Architectures Laboratory**

Electrical and Computer Engineering Department, MSN 1G5

George Mason University

Fairfax, VA - 22030

703.993.1774 (v)

703.993.1601 (f)

7. Email: **szaidi2@gmu.edu**

Regular Paper

This work was supported by the Air Force Office of Scientific Research (AFOSR) under Grants FA9550-05-1-0106 and FA9550-05-1-0388.

On Extending Temporal Models in Timed Influence Networks

Abbas K. Zaidi , Alexander H. Levis
System Architectures Lab
George Mason University
Fairfax, VA 22030
<szaidi2>, <alevis>@gmu.edu

P. Papantoni-Kazakos
Department of Electrical Engineering
University of Colorado,
Denver, CO 80202
Titsa.Papantoni@ucdenver.edu

Abstract

An extension to the formalism of Timed Influence Networks, including time-varying and cyclic influences is presented. The proposed time-varying influence functions allow modeling of influences whose strengths vary with time. A cyclic influence, on the other hand, provides a provision for a self-promoting set of influences. The two extensions will provide TIN models with the capability to depict PMESII (i.e., political, military, economic, social, infrastructure, and information) aspects of informal and uncertain domains, in the quest for the evaluation of various courses of action. An earlier mapping from a TIN model to a Time Sliced Bayesian Network is revisited and redefined for the new temporal extension. The temporal approach is illustrated with the help of a TIN model, used earlier for an effects based operations application.

1. Introduction

Complex decision problems, arising in areas ranging from financial markets to regional and global politics, often require modeling of informal, uncertain, and unstructured domains in order for a decision maker to evaluate alternatives and available courses of actions. The modeling of an uncertain domain using Probabilistic Belief Networks, or more commonly known as Bayesian Networks (BNs), is considered to be the most used and popular formalism. Influence Networks [6] are a variant of BNs that provide an intuitive and approximate language to elicit the large number of design parameters required for an underlying BN. The Influence Nets are especially useful for modeling situations in which it is difficult to fully specify all parameter values required for a BN and/or where their estimates are subjective e.g., when modeling potential human reactions and beliefs. Both Bayesian Networks and Influence Nets are designed to capture *static* interdependencies among variables in a system. A situation where the impact of a variable takes some *time* to reach the affected variable(s) cannot be modeled by either of the two approaches. In the last several years, efforts have been made to integrate the notion of time and uncertainty. Wagenhals et al. [14, 15, 17] have added a special set of temporal constructs to the basic formalism of Influence Nets. The Influence Nets with these additional temporal constructs are called Timed Influence Nets (TINs). TINs have been experimentally used in the area of Effects Based Operations (EBOs) for evaluating alternative courses of actions and their effectiveness to mission objectives in a variety of domains, e.g., war games [7, 8, 9, 16], and coalition peace operations [18], to name a few. The provision of time allows for the construction of alternate courses of action as timed sequences of actions or actionable events represented by nodes in a TIN [9, 15, 16]. A number of analysis tools have been developed over the years for TIN models to help an analyst update beliefs [3, 4, 5, 10, 11], represented as nodes in a TIN, to map a TIN model to a Time Sliced Bayesian Network for incorporating feedback evidence [4], to determine best course of actions for both timed and un-timed versions of Influence Nets [12], and to assess temporal aspects of the influences on objective nodes [20, 21].

In this paper, we present a further extension to the formalism of Timed Influence Networks with time-varying and cyclic influences. The proposed time-varying influence functions allow modeling of influences whose strengths vary with time. A cyclic influence, on the other hand, provides a provision for self-promoting set of influences. These two extensions will allow design of more realistic situations in the TIN models. The paper provides the theoretical description of the new temporal features within the TIN formalism. The temporal considerations are further explored and explained with the help of an experimental set up and its illustration with the help of example TIN models. The new temporal extension is applied to an earlier TIN model, used in the cited EBO applications above, and the results of the analyses are presented.

The rest of the paper is organized as follows: Section 2 provides a technical background of Timed Influence Nets. A detailed discussion on modeling temporal aspects of influence relationships is also provided in this section which includes the proposed time-varying influences and cyclic influences. In Section 3, an experimental set up is laid out for the analysis of a TIN model and some numerical results are presented with the help of examples. In Section 4 conclusions are drawn.

2. Timed Influence Networks

In a Timed Influence Network (TIN) setup, we are concerned with the evaluation of cause-effect relationships between interconnected events which may also have temporal delays associated with the cause-effect relationships. In particular, if the status of some event B is affected by the status of a set of events, A_1 to A_n, we are interested in a qualification, quantification, and timing of this effect. We first graph the relationships between events B and A_1 to A_n, also represented as $\{A_i\}_{1 \leq i \leq n}$. In a network format, as in Fig. 1 below, with each event being a node, with arcs indicating relationships and with arrows representing the cause-effect directions. The figure can be extended to a multi-layer graph to include evolutionary chains of inter-affecting events, where an affected event becomes an affecting event for some other event in the sequel. This graphical representation is identical to that used in BNs.

In this section, we provide a formal definition of Timed Influence Networks (INs), as presented in Zaidi et al [22]. This definition is adapted from the earlier definition of Influence Networks by Rosen and Smith [6]. In particular, we are interested in the effect the presence or absence of any of the events in the set $\{A_i\}_{1 \leq i \leq n}$ may have on the occurrence of event B, where the presence and absence of these events are also time-tagged.

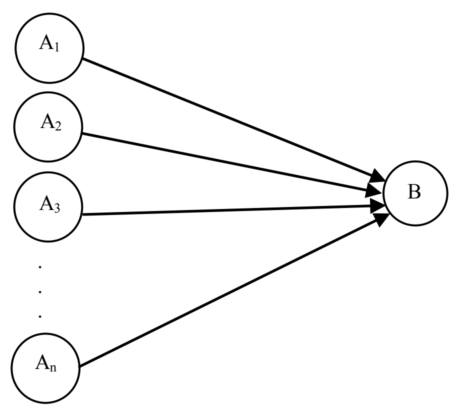


Fig. 1. Cause-Effect Relationships

Let us first define the terminology to be used, in the following description:

X_1^n: An n-dimensional binary random vector whose j^{th} component is denoted X_j, where $X_j = 1$; if the event A_j is present, and $X_j = 0$; if the event A_j is absent. We will denote by x_1^n realizations of the random vector c. A given realization x_1^n of the binary vector X_1^n describes precisely the status of the set $\{A_i\}_{1 \le i \le n}$ of events, regarding which events in the set are present. We name the vector X_1^n, the status vector of the affecting events.

To quantify the effects of the status vector X_1^n on the event B, we define the *influence function* $h_n(x_1^n)$ via the following qualitative properties:

$$h_n(x_1^n) = \begin{cases} 1 \quad ; \text{if} & \begin{array}{l}\text{given n affecting events, given the status} \\ \text{vector } x_1^n \text{, event B occours surely}\end{array} \\ -1 \quad ; \text{if} & \begin{array}{l}\text{given n affecting events, given the status} \\ \text{vector } x_1^n \text{, the nonoccurence of event B is sure}\end{array} \\ 0 \quad ; \text{if} & \begin{array}{l}\text{given n affecting events, given the status} \\ \text{vector } x_1^n \text{, the occurence of event B is unaffected}\end{array} \end{cases}$$

The definition in (1) is equivalent to (2) below, where $P(B \mid x_1^n)$ denotes the probability of occurrence of event B, given the status vector x_1^n.

$$P(B \mid x_1^n) = \begin{cases} 1 \quad ; h_n(x_1^n) = 1 \\ P(B); \ h_n(x_1^n) = 0 \\ 0 \quad ; h_n(x_1^n) = -1 \end{cases} \tag{2}$$

The definition of conditional probability in (2) is further extended in Zaidi et al [22] for all values in $[1,-1]$ of the influence function, via linear interpolation from (2) and the use of the unconditional probability $P(B)$. The definition is given in (3) below:

$$P(B \mid x_1^n) = \begin{cases} P(B) + h_n(x_1^n)[1 - P(B)]; \text{if } \ h_n(x_1^n) \in [0,1] \\ P(B) + h_n(x_1^n)P(B) \quad ; \text{if } h_n(x_1^n) \in [-1,0] \end{cases} \tag{3}$$

We note that there exist 2^n distinct values of the status vector x_1^n; thus, there exist 2^n distinct values of the influence function $h_n(x_1^n)$ as well as of the conditional probabilities in (4). In the case where there is only one affecting event, the influence function $h_1(x_1)$ has only two values, one for when $x_1 = 1$ and the other for when $x_1 = 0$. Note that the definition of this influence function only provides another way of looking at the aggregate influences otherwise captured as conditional probability values in a BN. The exponential number of values required to fully capture the influence on event B of all possible combinations of affecting events $\{A_i\}_{1 \le i \le n}$ poses the same ‗knowledge elicitation problem' of Bayesian Networks (BNs). In BNs, such knowledge is obtained via large data samples from stationary and ergodic environments. The Influence Networks defined below, eliminate the large data samples requirement of BNs, by first considering the influences of individual events in $\{A_i\}_{1 \le i \le n}$ on B in isolation, thus requiring only two values for a single-dimension influence function per each affecting event, i.e., $h_1(x_1)$. The formalism then utilizes several alternative methods to combine these single- dimension influence functions to construct higher dimension (i.e., n > 1) such functions. The higher dimension functions can then be used to elicit the conditional probabilities, as in (3).

Definition 1 *Tined Influence Network*

A Timed Influence Network (TIN) is a Bayesian Network mapping conditional probabilities $P(B \mid x_1^n)$ via the utilization of influence constants as in (3)**.** Formally, TIN is a tuple (**V**, **E**, **C**, **D**, $\mathbf{A_T}$, **B**) with **G** = (**V**, **E**) representing a *directed-acyclic* graph satisfying the Markov condition (as in BN), where

- **V**: set of nodes representing binary random variables,
- **E**: set of edges representing causal influences between nodes,
- **C**: set of causal strengths: $E \to \left\{ \left[h_1^{(i)}(x_i = 1), h_1^{(i)}(x_i = 0) \right] \text{such that } h_1\text{'s} \in [-1,1] \right\}$,
- **B**: Probability distribution of the status vector X_1^n corresponding to the external affecting events $\{A_i\}_{1 \le i \le n}$.
- **D**: set of temporal delays on edges: $\mathbf{E} \to \mathrm{N}$,
- $\mathbf{A_T}$: a subset of **V** representing *external* affecting events $\{A_i\}_{1 \le i \le n}$ and a status of the corresponding vector X_1^n. The status of each external affecting event is *time tagged* representing the time of realization of its status. In the TIN literature [9, 14, 15, 16, 18, 13, 19], A_T is also referred to as a Course of Action (COA). A COA is, therefore, a time-sequenced collection of external affecting events and their status.

We now proceed with a definition which will lead to a mathematically correct relationship between influence functions and unconditional probabilities.

A TIN is called *consistent* if it observes the Bayes' Rule, as presented in Lemma 1.

Lemma 1 *Consistency Condition* [22]
Let the influence function $h_n(x_1^n)$ be accepted as reflecting accurately the causal relationship between the affecting events $\{A_i\}_{1 \le i \le n}$ and event B. Then the TIN model is consistent iff:

$$P(B) = \left[\sum_{x_1^n : \mathrm{sgn} h_n(x_1^n) = 1} P(x_1^n) h_n(x_1^n) \right] \left[\sum_{x_1^n} P(x_1^n) \left| h_n(x_1^n) \right| \right]^{-1} \text{; when } \sum_{x_1^n} P(x_1^n) \left| h_n(x_1^n) \right| \neq 0 \qquad (4)$$

2.1 Influence Functions

In this section, we list some specific influence functions, $h_n(x_1^n)$, first presented in Zaidi et al [22]. They are specific analytic functions of the one-dimensional components $h_1^{(i)}(x_i)$; $1 \le i \le n$ as described in Definition 1. It should be noted that we are not mapping the $\{h_1^{(i)}(x_i)\}_{1 \le i \le n}$ values onto conditional probabilities $\{P(B \mid x_i)\}_{1 \le i \le n}$. Instead, we are using the values for $\{h_1^{(i)}(x_i)\}_{1 \le i \le n}$ to construct a global $h_n(x_1^n)$ influence function; it is the latter function which is mapped onto the conditional probability $P(B \mid x_1^n)$, as in (3). The analytical functions presented below can be categorized into *multiplicative* and *additive* models due to the way these one-dimensional components are combined to form the higher-order influence function.

A. Multiplicative Models

1) The $h_n(x_1^n)$ *Corresponding to the CAST logic* [6]

The influence constant presented below is that used by the CAST logic in [6].

In this case, given the values for $\{h_1^{(i)}(x_i)\}_{1\le i\le n}$ the global influence function, $h_n(x_1^n)$, is defined as follows:

$$h_n(x_1^n)=\left[\prod_{i:h_1(x_i)<0}\left(1-\left|h_1^{(i)}(x_i)\right|\right)-\prod_{i:h_1(x_i)>0}\left(1-\left|h_1^{(i)}(x_i)\right|\right)\right]\times\left[\max\left(\prod_{i:h_1(x_i)<0}\left(1-\left|h_1^{(i)}(x_i)\right|\right),\prod_{i:h_1(x_i)>0}\left(1-\left|h_1^{(i)}(x_i)\right|\right)\right)\right]^{-1} \tag{5}$$

The global values of the function $h_n(x_1^n)$ and the probabilities $P(x_1^n)$ and $P(B)$ must satisfy the consistency condition as in (4). We, therefore, determine $P(B)$ via (4) and the conditional probabilities $P(B\,|\,x_1^n)$ are then calculated via (3).

2) The $h_n(x_1^n)$ *Representing Noisy OR Format* [1, 2]

Given the constants $\{h_1^{(i)}(x_i)\}_{1\le i\le n}$, we define here $h_n(x_1^n)$ as follows; where α is such that $0\le\alpha\le1$:

$$h_n(x_1^n)=\left\{1-(1-\alpha)^{-1}\prod_{i=1}^{n}\left(1-\left|h_1^{(i)}(x_i)\right|\right)\right\}^{\operatorname{sgn}\left(h_n(x_1^n)\right)}\left\{-1+\alpha^{-1}-\alpha^{-1}\prod_{i=1}^{n}\left(1-\left|h_1^{(i)}(x_i)\right|\right)\right\}^{1-\operatorname{sgn}\left(h_n(x_1^n)\right)} \tag{6}$$

Then, via (4), we obtain:

$$P(B)=\alpha$$

$$1-P(B\,|\,x_1^n)=\prod_{i=1}^{n}\left(1-\left|h_1^{(i)}(x_i)\right|\right)$$

The expression in (6) represents the Noisy-OR format [1, 2], where the probabilities in the latter are here substituted by the absolute values of the one-dimensional influence functions $\{h_1^{(i)}(x_i)\}_{1\le i\le n}$.

B. Additive Model

3) A Linear $h_n(x_1^n)$

In this case, we assume that the effects of events $\{A_i\}_{1\le i\le n}$ on event B are weighted by a known set $\{w_i\}_{1\le i\le n}$ of weights, such that $w_i\ge0;\forall i$ and $\sum_{i=1}^{n}w_i=1$. Then, given the constants $\{h_1^{(i)}(x_i)\}_{1\le i\le n}$, we define $h_n(x_1^n)$ as follows, for some given value $\alpha: 0\le\alpha<1$:

$$h_n(x_1^n)=\begin{cases}(1-\alpha)^{-1}\sum_{i=1}^{n}w_ih_1^{(i)}(x_i) & ;\quad \left|\sum_{i=1}^{n}w_ih_1^{(i)}(x_i)\right|\le1-\alpha\\ 1 & ;\quad \sum_{i=1}^{n}w_ih_1^{(i)}(x_i)\ge1-\alpha\\ -1 & ;\quad \sum_{i=1}^{n}w_ih_1^{(i)}(x_i)\le-(1-\alpha)\end{cases} \tag{7}$$

A nonzero α value translates to the probability of event B being equal to one, not only when all the

$\{h_1^{(i)}(x_i)\}_{1\le i\le n}$ values equal one, but also when a predefined weighted majority exceeds a total weighted sum of $1-\alpha$. Similarly then, the event B occurs with zero probability when the weighted sum of the $\{h_1^{(i)}(x_i)\}_{1\le i\le n}$ values is less than $-(1-\alpha)$, rather than only when it equals -1.

2.1 Temporal Evolution of Influences

In this section, we present the formalization of the temporal issues involved in the development of TINs. In particular, we present the dynamics of the $\{A_i\}_{1\le i\le n}$ to B relationship, when the status of various affecting events are learned asynchronously in time. This asynchronous evolution of influences is due to the temporal parameters defined in the definition of TINs. The time tags on the affecting events $\{A_i\}_{1\le i\le n}$ correspond to the time points at which the status of these events is realized. The delay parameters on edges, on the other hand, determine the time taken by an influence of an affecting event to reach B. An algorithm by Haider and Zaidi [4] determines a time sequence for the realization of affecting events' status with the help of time tags on affecting events and draws a *time-sliced* version of a TIN after taking into account the delays on edges. Fig. 2 shows an example TIN with time-tagged affecting events and with delays on the edges. Fig. 3 shows the resulting time-sliced TIN obtained by the application of algorithm in [4] on the TIN in Fig. 2. We assume in our description that $\{A_i\}_{1\le i\le n}$ is the maximum set of events affecting event B. It is evident from Fig. 3 that there might be time instances when the status of some of the affecting events may be unknown. Without lack in generality – to avoid cumbersome notation – we also assume that the affecting events $\{A_i\}_{1\le i\le n}$ are ordered in the order when their status become known. That is, the status of events A_i is first known, then that of event A_2, and so on. Towards this direction, we derive a dynamic programming relationship between the influence functions $h_n(x_1^n)$ and $h_{n-1}(x_1^{n-1})$, where $h_{n-1}(x_1^{n-1})$ corresponds to the case where the status of the affecting event A_n is unknown.

Lemma 2 [22]
Let the joint probability $P(x_1^n)$ be given and the probability $P(B)$ is defined as in (4) and let $P(x_n \mid x_1^{n-1})$ denote the probability of the least bit in the status vector X_1^n being x_n, given that the reduced status vector value is x_1^{n-1}. Then, $h_{n-1}(x_1^{n-1})$ is given as a function of $h_n(x_1^n)$, as shown below.

$$h_{n-1}(x_1^{n-1}) = \begin{cases} Q_n & ; \quad Q_n \in [-1,0] \\ P(B)[1-P(B)]^{-1}Q_n & ; \quad Q_n \in [0, P^{-1}(B)-1] \end{cases} \tag{8}$$

where

$$Q_n \stackrel{\Delta}{=} \sum_{x_n=0,1} P(x_n \mid x_1^{n-1})\{h_n(x_1^n)\}\{[1-P(B)]P^{-1}(B)\}^{\mathrm{sgn}\, h_x(x_1^n)}$$

We note that the influence functions are deduced from the same functions of higher dimensionality, as shown in Lemma 2. In accordance, conditional probabilities of event B are produced from the deduced influence function values, via (3), as:

$$P(B \mid x_1^{n-1}) = P(B)\{1 + h_{n-1}(x_1^{n-1})[1-P(B)]P^{-1}(B)\}^{\mathrm{sgn} h_{n-1}(x_1^{n-1})} \bullet \{1 + h_{n-1}(x_1^{n-1})\}^{1-\mathrm{sgn} h_{n-1}(x_1^{n-1})}$$

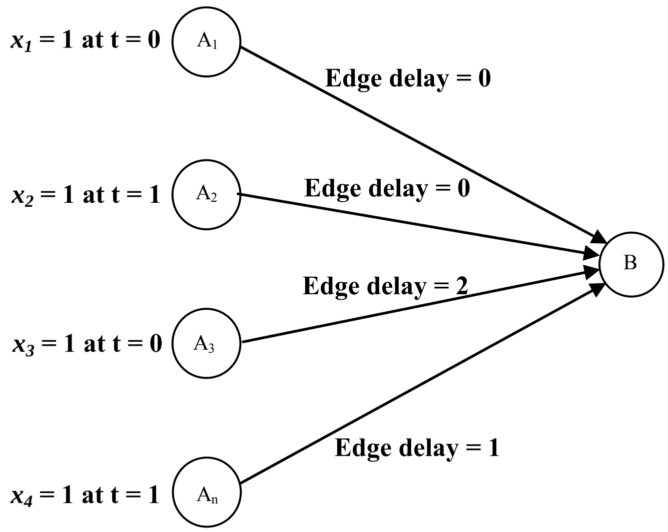


Fig. 2. Example TIN with COA and Edge Delays

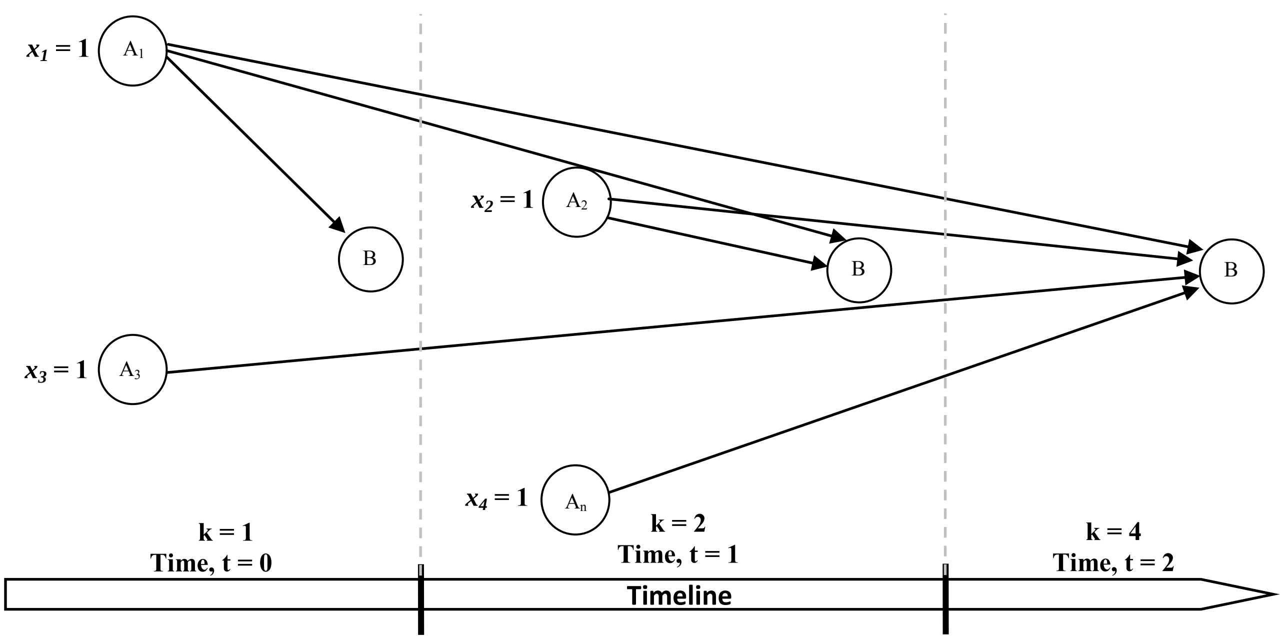


Fig. 3. Temporal Model for the Example TIN in Fig. 2.

In an attempt to formalize this temporal evolution, let T_0 denote the time when the computation of the system dynamics starts. Let T_1 denote the time when the status of event A_1, becomes known. Let T_k; $1 \leq k \leq n$ denote the time when the status of event A_k becomes known. Then at time T_k, the conditional probabilities $P(B \mid x_1^k)$ is computed as:

$$P\left(B \mid x_1^k\right)_{i \leq k \leq n} = P(B)\left\{1 + h_k(x_1^k)\left[1 - P(B)\right]P^{-1}(B)\right\}^{\mathrm{sgn}\, h_k(x_1^k)} \left\{1 + h_k(x_1^k)\right\}^{1-\mathrm{sgn}\, h_k(x_1^k)} \tag{9}$$

; where the probability $P(B)$ is computed via the consistency condition (4).

The conditional probabilities of event B evolve dynamically and finally converge to the probability $P(B \mid x_1^n)$ at time T_n, when the status of all the affecting events becomes known. It is important to point out that the conditional probability in (9) is sensitive to the time ordering of the affecting events. That is, the probability of the decisive event B, given the partial status vector x_1^k is defined by the time ordering of the affecting events at time k.

Another temporal situation where the existence as well as the status of the affecting events are sequentially revealed, then at time k, $P_k(B)$ and $P_k(B \mid x_1^k)$ are computed as in (4) and (9) where n is substituted by k in the former and $P(B)$ is replaced by $P_k(B)$ in both. Computationally, the difference between the two situations is the evolution of $P_k(B)$ with the revelation of affecting events. We note that the time evolution of the conditional probabilities $P_k(B \mid x_1^k)$ for this temporal situation is different for different time orderings of the affecting events $\{A_i\}_{1 \le i \le n}$ presented earlier.

2.2 Time-varying Influences

In this section, we take the definition of influence functions a step further and propose time-varying influences that take on different values at different time points in the time-sliced version of a TIN. In the previous definition of influence functions the temporal parameters may determine the number of known/unknown affecting events at some time point and the aggregate influence captured by the value of an influence function at that time is determined only by the number of affecting events with known states. We now present influences that are also functions of time. This time-varying nature of influences is reflected by time-varying influence functions $h_n(x_1^n)$. For each given such function, as in (5), (6), and (7) that have the form:

$$h_n(x_1^n) = f_n(\{h_1^{(i)}(x_i)\} \quad ; \quad 1 \le i \le n)$$

The time varying property can be imposed on each $h_1^{(i)}(x_i)$ component by the use of following choices:

$$f(h_1^{(i)}(x_i),t) \rightarrow h_1^{(i)}(x_i)[u(t)-u(t+\gamma)]$$

$$\text{where, } \gamma : 0 < \gamma \text{ and } u(x) = \begin{cases} 1 & ; \quad x \ge 0 \\ 0 & ; \quad x < 0 \end{cases} \tag{10}$$

or,

$$f(h_1^{(i)}(x_i),t) \rightarrow h_1^{(i)}(x_i) \;\; e^{-\alpha(t-t_0)} \quad \text{where, } \alpha > 0, \;\; t_0 > 0 \tag{11}$$

For more tractable analysis and implementation, we may impose the time-varying property on the overall influence functions $h_n(x_1^n)$ as follows:

$$f(h_n(x_1^n),t) = h_n(x_1^n)[u(t)-u(t+\gamma)] \tag{12}$$

where,

$$\gamma > 0 \qquad u(x) = \begin{cases} 1 & ; \quad x \ge 0 \\ 0 & ; \quad x < 0 \end{cases}$$

or,

$$f(h_n(x_1^n),t) = h_n(x_1^n)\ \ e^{-\alpha(t-t_0)} \text{ where, } \alpha > 0,\ \ t_0 > 0 \tag{13}$$

Then, the consistency condition remains unchanged. That is, $P(B)$ is found (without the time element injected) from:

$$\sum_{X_1^n} P(x_1^n)\{1 + h_n(x_1^n)[1 - P(B)]\ \ P^{-1}(B)\}^{\operatorname{sgn} h_n(x_1^n)} .\{1 + h_n(x_1^n)\}^{1-\operatorname{sgn} h_n(x_1^n)} = 1$$

The conditional probabilities and the evolving lower dimensionality influence functions will now be time varying and are determined by the following:

$$P_t(B \mid x_1^n) = P(B)\ \{1 + f(h_n(x_1^n),t)[1 - P(B)]\ \ P^{-1}(B)\}^{\operatorname{sgn} f(h_n(x_1),t)} .\{1 + f(h_n(x_1^n),t)\}^{1-\operatorname{sgn} f(h_n(x_1^n),t)}$$

$$f(h_{n-1}(x_1^{n-1}),t) = \begin{cases} Q_n^{(t)} - 1 \ \ ; \ \ Q_n^{(t)} \in [0,1] \\ P(B)[1 - P(B)]^{-1}[Q_n^{(t)} - 1] \ \ ; \ \ Q_n^{(t)} \in [1, P^{-1}(B)] \end{cases} \tag{14}$$

where, $Q_n^{(t)} \overset{\Delta}{=}$

$$\sum_{X_{n=0,1}} P(x_n \mid x_1^{n-1})\{1 + f(h_n(x_1^n),t)[1 - P(B)]\ \ P^{-1}(B)\}^{\operatorname{sgn} f(h_n(x_1^n),t)} .\{1 + f(h_n(x_1^n),t)\}^{1-\operatorname{sgn} f(h_n(x_1^n),t)} = 1$$

This time-varying nature of influences is illustrated in Fig. 4. The figure shows the time-sliced version of the TIN in Fig. 2 with time-varying influence components $h_1^1(1) = 0.99\ e^{-0.2t}$, $h_1^1(0) = -0.99\ e^{-0.2t}$ on the edge connecting affecting event A_1 to node B.

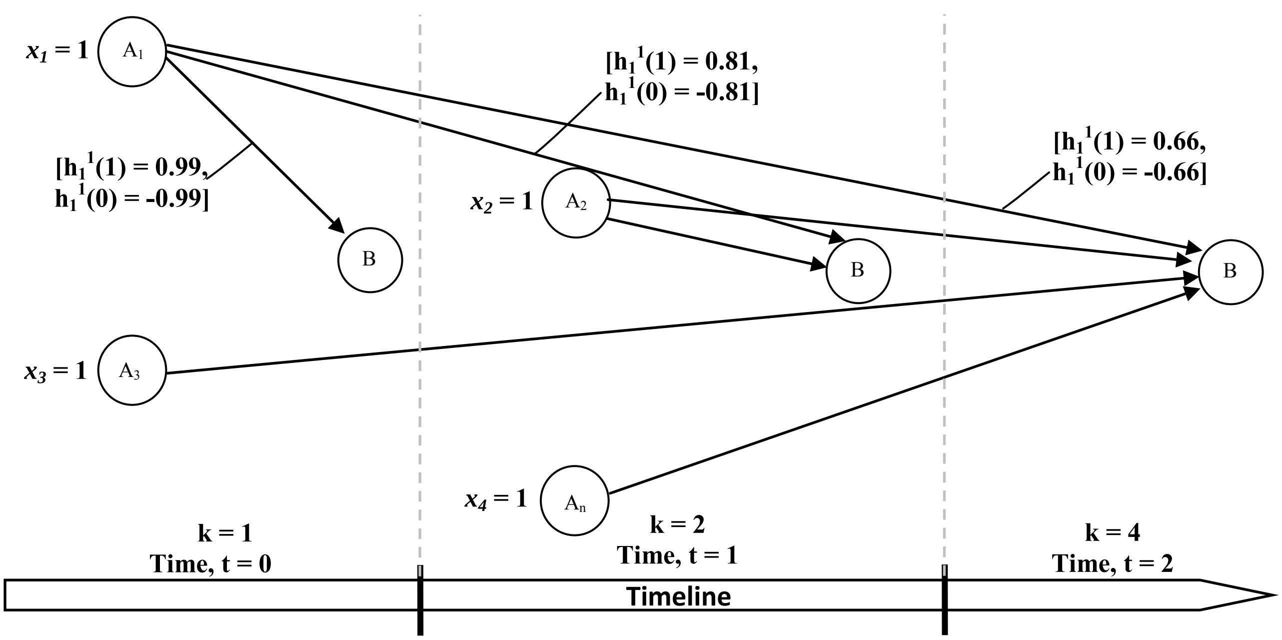


Fig. 4. Temporal Model for the Example TIN in Fig. 2 with Time-varying influence $\mathbf{h_1^1(1) = 0.99\ e^{-0.2t}}$, $\mathbf{h_1^1(0) = -0.99\ e^{-0.2t}}$ on the edge from A_1 to effect node B.

2.3 Cyclic Influences

In this section, we propose to relax the condition in the definition of a Timed Influence Nets which requires the graphical representation of a TIN to be a directed-acyclic graph, i.e, no loops. In the temporal evolution of influences in TINs, as illustrated with the TIN in Fig. 2, and its time-sliced version in Figs. 3 and 4, one can easily observe that if cycles of non-zero delays (i.e., the sum of delays on edges involved in a cycle) are permitted in a TIN, then such cycles will disappear in the time-sliced version of that TIN. A cycle in a TIN will unravel its constituent nodes on subsequent time slices without an influence ever going back in time to create a cycle in the time-sliced case. The key requirement, however, is to have these cycles with non-zero positive delays. The resulting time-sliced TIN will conform to the acyclic graph requirement and, therefore, will be amenable for analysis as presented in the following sections. The algorithm by Haider and Zaidi [4] that transforms a TIN into its time-sliced representation requires a minor modification to accommodate the presence of cycles in the original TIN. In the new implementation of this algorithm, the cycles are first identified as *strongly-connected components* in a TIN graph. The sub-graphs representing these strongly-connected components are replaced by single nodes, thus removing cycles in the graphical representation. The old algorithm then estimates the maximum number of slices required for a complete analysis of influences in the time sliced TIN. The new approach adds to this maximum the delays associated with the cycles. Once the maximum number of time slices is determined, the TIN is transformed to its time-sliced representation using the approach in [4]. Fig. 5 presents a TIN model with a cycle in it. A time-sliced version of this TIN is shown in Fig. 6, obtained via the modified algorithm. It can be seen in the TIN of Fig. 6 that the cyclic influence in Fig. 5 unravels itself on the timeline.

The provision of cyclic influences in a TIN is a useful construct in modeling self-promoting set of influences where the status of an affecting event may in turn be influenced by an affected event.

3. Experimental Setup and Numerical Evaluations

In this section, we lay out the steps involved in an experimental setup.

a. Given an event B, determine *all* the events $\{A_i\}_{1 \le i \le n}$ *known* to be affecting its occurrence.
b. Given B, all the known affecting events $\{A_i\}_{1 \le i \le n}$, and the causal strengths $\left[h_1^{(i)}(x_i = 1), h_1^{(i)}(x_i = 0)\right]$ between each A_i and B, design an influence constant $h_n(x_1^n)$, where x_1^n signifies the value of the status vector of the events $\{A_i\}_{1 \le i \le n}$, and where $-1 \le h_n(x_1^n) \le 1; \forall\, x_1^n$ values. The $h_n(x_1^n)$ constant may have one of the forms presented in section IX.
c. If *all* in (b) is given, then upon a given probability of the status vector X_1^n, say $P(x_1^n); \forall\, x_1^n$ values, the probability of event B is given by consistency equation in (4)
d. When *all* the affecting events $\{A_i\}_{1 \le i \le n}$ are known, but the status of some of them are unknown, then, the probability $P(B)$, as computed in step (c) is used to compute the conditional probability $P(B \mid x_1^k)$ as given in (9), where k represents the known affecting events and $h_k(x_1^k)$ is computed in a dynamic programming fashion from the influence constant $h_n(x_1^n)$ in (b). Note that the affecting events $\{A_i\}_{1 \le i \le n}$ are assumed ordered as of the revealing of their status in time. Different such ordering results in different evolutions of the conditional probabilities $P(B \mid x_1^k)$.
e. When the existence as well as the status of the affecting events are sequentially revealed, then at time

k, $P_k(B)$ and $P_k(B \mid x_1^k)$ are computed as in (c) and (d) where n is substituted by k in the latter.

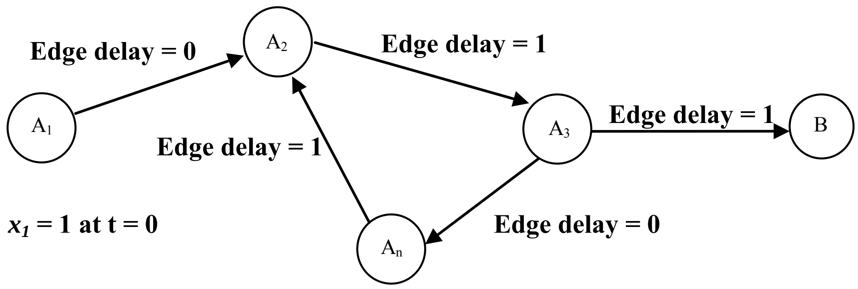


Fig. 5. Example TIN with Cyclic Influences

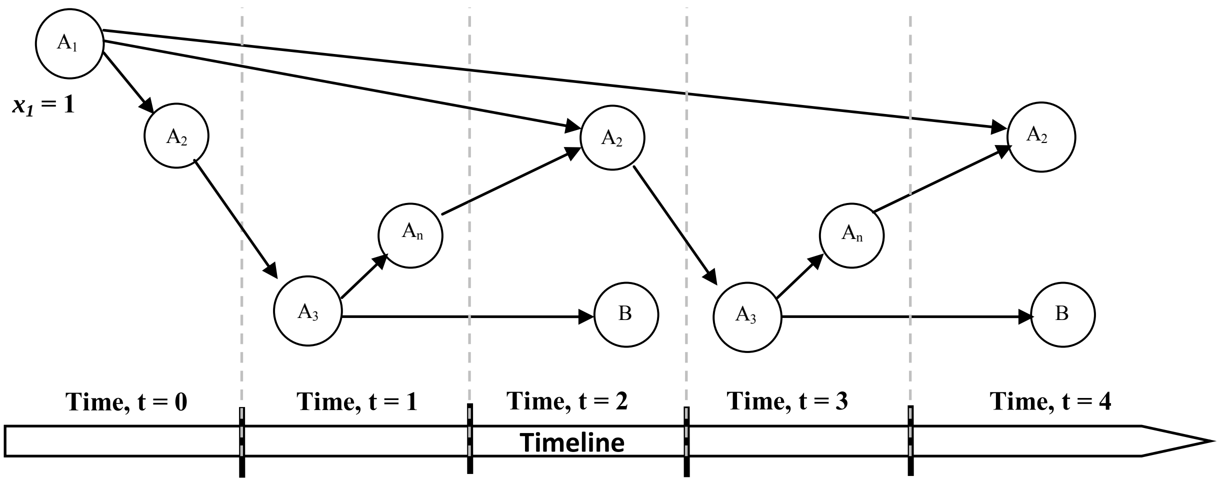


Fig. 6. Temporal Model for the Example TIN in Fig. 5.

Example 1:

The following example illustrates the steps (a) to (e) with the help of the TIN in Figs. 2 and 3.

i. Fig. 7 shows the TIN in Fig. 2 annotated with the constants $\left[h_1^{(i)}(x_i = 1), h_1^{(i)}(x_i = 0)\right]$ for each i, where $x_i = 0, 1$ represents one of the two states of an affecting event A_i. A global influence function $h_4(x_1^4)$ is then designed using the multiplicative model 1 in Section 2.1. Table 1 shows the computed values for $h_4(x_1^4)$; $\forall\, x_1^n$.

ii. The joint probability $P(x_1^4)$; $\forall\, x_1^4$ values are computed by assigning $P(x_i = 1) = P(x_i = 0) = 0.5$; $\forall\, i$ and by assuming $\{A_i\}_{1 \le i \le 4}$ to be mutually independent. The probability of occurrence of event B, i.e., z = 1, is now calculated with the consistency equation, and is given as $P(z = 1) = 0.5$. Assuming the status of all the affecting events to be known, the conditional probabilities $P(B \mid x_1^4); \forall x_1^4$ are calculated via (9), and are shown in Table 1.

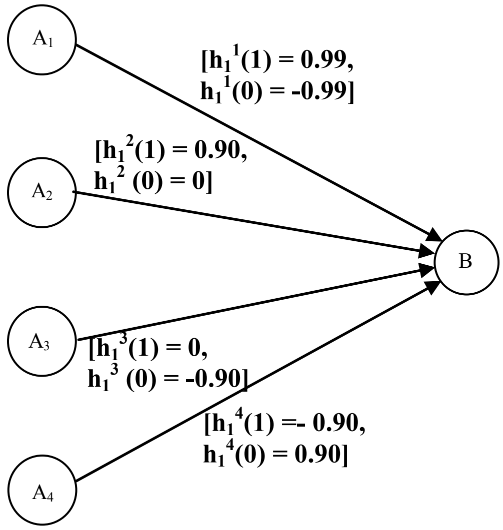


Fig. 7. Example TIN

TABLE 1

x_1	x_2	x_3	x_4	$h_4(x_1^4)$	$P(z=1 \mid x_1^4)$
0	0	0	0	-0.990000	0.005000
0	0	0	1	-0.999900	0.000050
0	0	1	0	-0.900000	0.050000
0	0	1	1	-0.999000	0.000500
0	1	0	0	-0.900000	0.050000
0	1	0	1	-0.999000	0.000500
0	1	1	0	-0.000001	0.499999
0	1	1	1	-0.990000	0.005000
1	0	0	0	0.990000	0.995000
1	0	0	1	0.000001	0.500001
1	0	1	0	0.999000	0.999500
1	0	1	1	0.900000	0.950000
1	1	0	0	0.999000	0.999500
1	1	0	1	0.900000	0.950000
1	1	1	0	0.999900	0.999950
1	1	1	1	0.990000	**0.995000**

iii. The assumption in step (ii), regarding the knowledge of the status of all the affecting events, may not be valid at times. Such is the case of a TIN with delays on edges, reflecting variations in the times when the status of the affecting events become known (see Definition 1). To illustrate this notion, we use the TIN in Fig.2. The time assigned to an affecting event A_i is the instance at when it assumes a state, i.e., $x_i = 0$ or 1. Prior to that time, the state of the event is assumed unknown. As stated in Definition 1, this combination of the external affecting events' status and their timing is also termed a Course of Action (COA), in the TIN literature.

iv. The temporal information in the TIN, Fig. 2, determines the dynamics of the relationship between the affecting events and the affected event B; specifically, the times when the status of the affecting events are revealed to B. Fig. 3 shows a time-sliced version of this TIN, obtained by mapping the status of the affecting events and their effects on the event B, on a timeline. This mapping determines the number of affecting events ‗k‘ at different time points (or time slices). For the temporal case presented above, the existence of all the affecting events is known to the event B a priori; their status, however, remain unknown until revealed, as determined by the COA (i.e., time-tagged set of external affecting events) and the delays on the edges. The probability $P(B)$, as calculated in step (c), is used to compute the conditional probabilities $P(B \mid x_1^k); k = 1,2,4$, i.e., $P(B \mid x_1^1)$, $P(B \mid x_1^2)$, and $P(B \mid x_1^4)$, as illustrated in the figure. Table 2 shows the values for $P(B \mid x_1^1)$ and $P(B \mid x_1^2)$, as computed by the corresponding $h_1(x_1^1)$ and $h_2(x_1^2)$. The posterior probability of B captures the impact of an affecting event on B and can be plotted as a function of time for a corresponding COA. This plot is called a Probability Profile [13, 14].

TABLE 2

x_1	$P(z=1 \mid x_1^1)$	x_1	x_2	$P(z=1 \mid x_1^2)$
0	0.076381	0	0	0.013887
1	**0.923619**	0	1	0.138875
		1	0	0.861125
		1	1	**0.986113**

A. Probability Propagation in a Multi-node Network

In multi-node connected network structures, given a set of external affecting events $\{A_i\}_i$, given influence constants $\{h_n(x_1^k)\}_k$, pertinent conditional probabilities are constructed hierarchically, as the structure of the network dictates. Consider, for example, the network in Fig. 8. In this network, the affecting events $A_i; i = 1,2,3,4$ are external and unaffected by other events, while events B and C are affected, B being affecting as well. Let us denote the status of event $A_i; i = 1,2,3,4$; x_i, the status of event B as y and the status of event C as z, where y, z and $\{x_i\}_{1 \le i \le 4}$ are 0-1 binary numbers. Let the influence functions $h(x_1, x_2), h(x_3, x_4)$ and $h(y, x_3, x_4)$ be given. Let also the joint probability $P(x_1, x_2, x_3, x_4)$ be given. We then compute all the pertinent probabilities in the above network following the steps stated below:

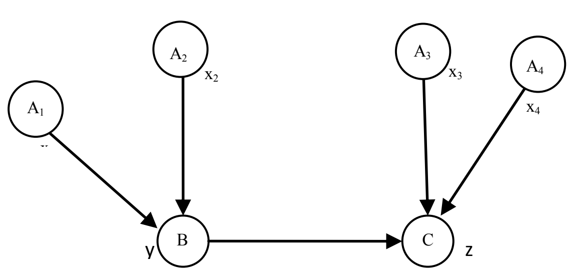


Fig. 5. A Multi-node Influence Network

1. Compute the probability $P(y)$ from the consistency condition:

$$\sum_{x_1,x_2} P(x_1,x_2)\{1+h(x_1,x_2)[1-P(y)]P^{-1}(y)\}^{\mathrm{sgn}h(x_1,x_2)}\{1+h(x_1,x_2)\}^{1-\mathrm{sgn}h(x_1,x_2)}=1$$

2. Compute $P(y\mid x_1,x_2)$ from, $P(y\mid x_1,x_2)=P(y)\{1+h(x_1,x_2)[1-P(y)]P^{-1}(y)\}^{\mathrm{sgn}h(x_1,x_2)}\{1+h(x_1,x_2)\}^{1-\mathrm{sgn}h(x_1,x_2)}$
3. Compute $P(y,x_3,x_4)$ as: $P(y,x_3,x_4)=\sum_{x_1,x_2} P(y\mid x_1,x_2)P(x_1,x_2,x_3,x_4)$
4. Compute $P(z)$ from the consistency condition

$$\sum_{y,x_3,x_4} P(y,x_3,x_4)\{1+h(y,x_3,x_4)[1-P(z)]P^{-1}(z)\}^{\mathrm{sgn}h(y,x_3,x_4)}\{1+h(y,x_3,x_4)\}^{1-\mathrm{sgn}h(y,x_3,x_4)}=1$$

5. Compute $P(z\mid y,x_3,x_4)$ from,

$$P(z\mid y,x_3,x_4)=P(z)\{1+h(y,x_3,x_4)[1-P(z)]P^{-1}(z)\}^{\mathrm{sgn}h(y,x_3,x_4)}\{1+h(y,x_3,x_4)\}^{1-\mathrm{sgn}h(y,x_3,x_4)}$$

6. Compute $P(z\mid x_1,x_2,x_3,x_4)$ from,

$$P(z\mid x_1,x_2,x_3,x_4)=\sum_{y} P(z,y\mid x_1,x_2,x_3,x_4)=\sum_{y} P(z\mid y,x_3,x_4)P(y,\mid x_1,x_2)$$

B. A Hypothetical Illustrative Example

In this section, we apply some of the presented temporal aspects in this paper to an illustrative TIN. The model used in this section was first presented by Wagenhals et al. in 2001 [18] to address the following scenario: As described in [18], internal political instabilities in Indonesia have deteriorated and ethnic tensions between the multiple groups that comprise Indonesia have increased. Religion has been a major factor in these conflicts. Members of one of the minority (2%) religious groups have banded together to combat disenfranchisement. These members have formed a rebel militia group. Armed conflicts recently occurred between those rebels and the Indonesian military. The rebels fled to eastern Java where they have secured an enclave of land. This has resulted in a large number of Indonesian citizens being within the rebel-secured territory. Many of these people are unsympathetic to the rebels and are considered to be at risk. It is feared that they may be used as hostages if ongoing negotiations break down with the Indonesian government. The food and water supply and sanitation facilities are very limited within the rebel-secured territory.

Several humanitarian assistance (HA) organizations are on the island, having been involved with food distribution and the delivery of public health services to the urban poor for several years. So far, the rebels have not prevented HA personnel from entering the territory to take supplies to the citizens. The U.S. and Australian embassies in Jakarta are closely monitoring the situation for any indications of increasing rebel activity. In addition, Thailand, which has sent several hundred citizens to staff numerous capital investment projects on Java, is known to be closely monitoring the situation. To reflect the situation stated above, a TIN was first created in [18] and is shown in Fig. 9.

The latter TIN models the causal and influencing relationships between (external) affecting events (on the left side and along the top of the model in Fig. 9) and the overall effect of concern which is the single node with no children on the right-hand side of the model. In this case, the effect is ―Rebels decide to avoid violence". The actionable (external) events in this model include a combination of potential coalition, UN, and rebel actions. The coalition actions include actions by the US government, its military instrument of national power, actions by the Government of Indonesia, and actions by Thailand.

For the purpose of illustration, we have selected a part of this network, as shown in Fig. 10. For the sake of brevity, the influences used in this illustrative example are all static and acyclic. The presence of time-varying and cyclic influences only changes the way temporal (i.e., time-sliced) model evolves over time and the values of the influences that are used to estimate the conditional probabilities (as illustrated in

Figs. 4-6). Once a temporal model is determined, the probability propagation is carried out using the steps presented in Section 3A.

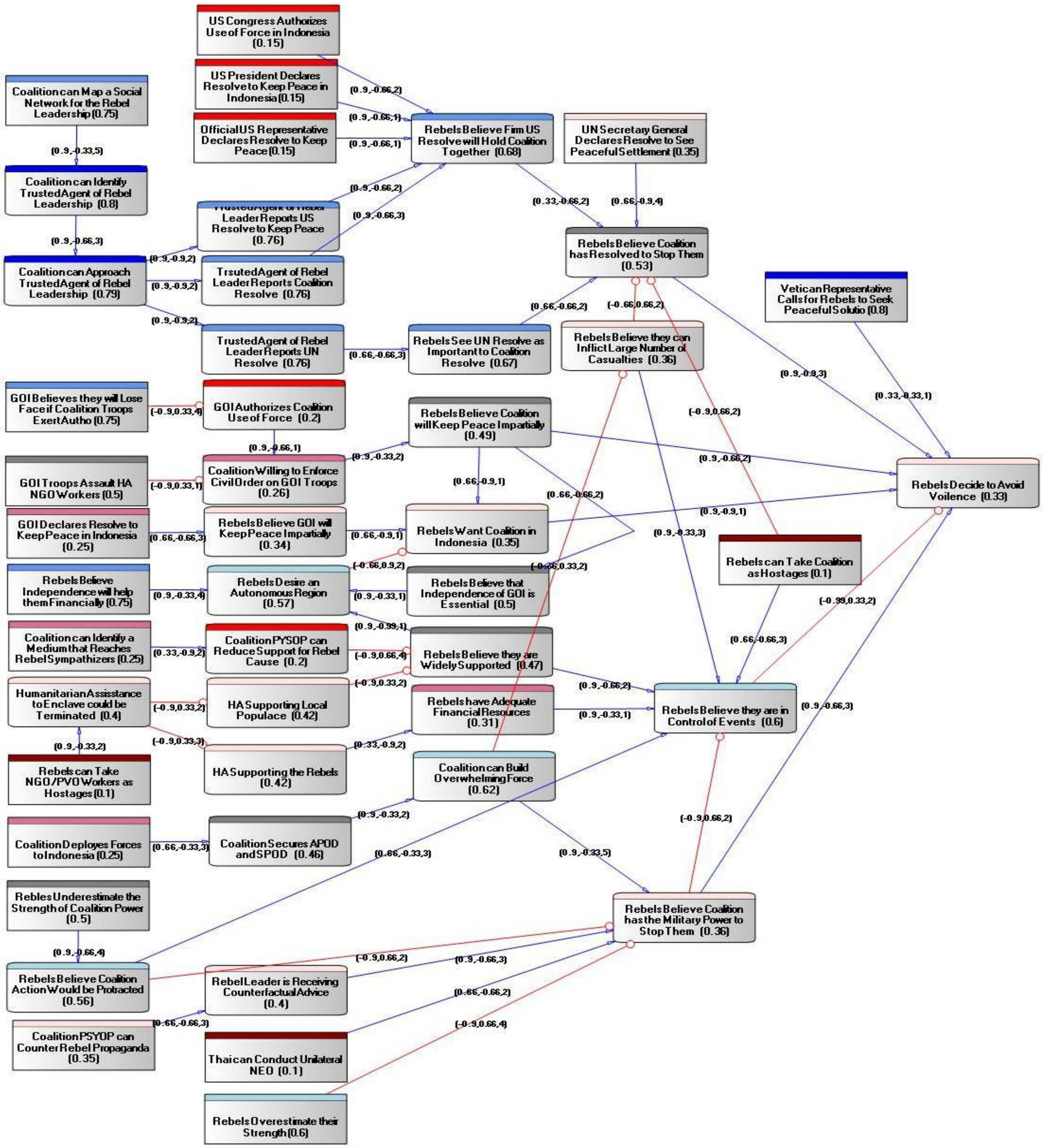


Fig. 9. Timed Influence Net Inspired by the East Timor Situation [18]

The (external) affecting events in the TIN of Fig. 10 are drawn as root nodes (nodes without incoming edges). The text in each node, e.g., —1—Coalition Deploys Forces to Indonesia," represents a node ID and a statement describing the binary proposition. In Fig. 11, $\{A_i\}_{0 \le i \le 4}$ represents the set of the external affecting events, where the index _i‘ depicts the node ID. The marginal probabilities for the external affecting events are also shown inside each node. In this illustration, we assume all external affecting events to be mutually independent (Section IV.) A desired effect, or an objective which a decision maker is interested in, is modeled as a leaf node (node without outgoing edges). The node with ID _10‘ in Fig. 10 represents the objective for the illustration. In both Figs. 6 and 7, the root nodes are drawn as rectangles while the non-root nodes are drawn as rounded rectangles. A directed edge with an arrowhead between two nodes shows the parent node promoting the chances of a child node being true, while the roundhead edge shows the parent node inhibiting the chances of a child node being true. The first two elements in the inscription associated with each arc quantify the corresponding strengths of the influence of a parent node‘s state (as being either true or false) on its child node. The third element in the inscription depicts the time it takes for a parent node to influence a child node. For instance, in Fig. 10, event ―1‖Coalition Deploys Forces to Indonesia” influences the occurrence of event ―7‖Coalition Secures APOD and SPOD” after 3 time units.

The purpose of building a TIN is to evaluate and compare the performances of alternative courses of actions described by the set $\mathbf{A_T}$ in the definition of TINs. The impact of a selected course of action on the desired effect is analyzed with the help of a probability profile. The following is an illustration of such an analysis with the help of two COAs, given below:

<u>COA1</u>: *All external affecting events are taken simultaneously at time 1 and are mutually independent.*

<u>COA2</u>: *Events {0, 2, 4} are taken at time 1, simultaneously, and events {1, 3} are taken at time 2, simultaneously.*

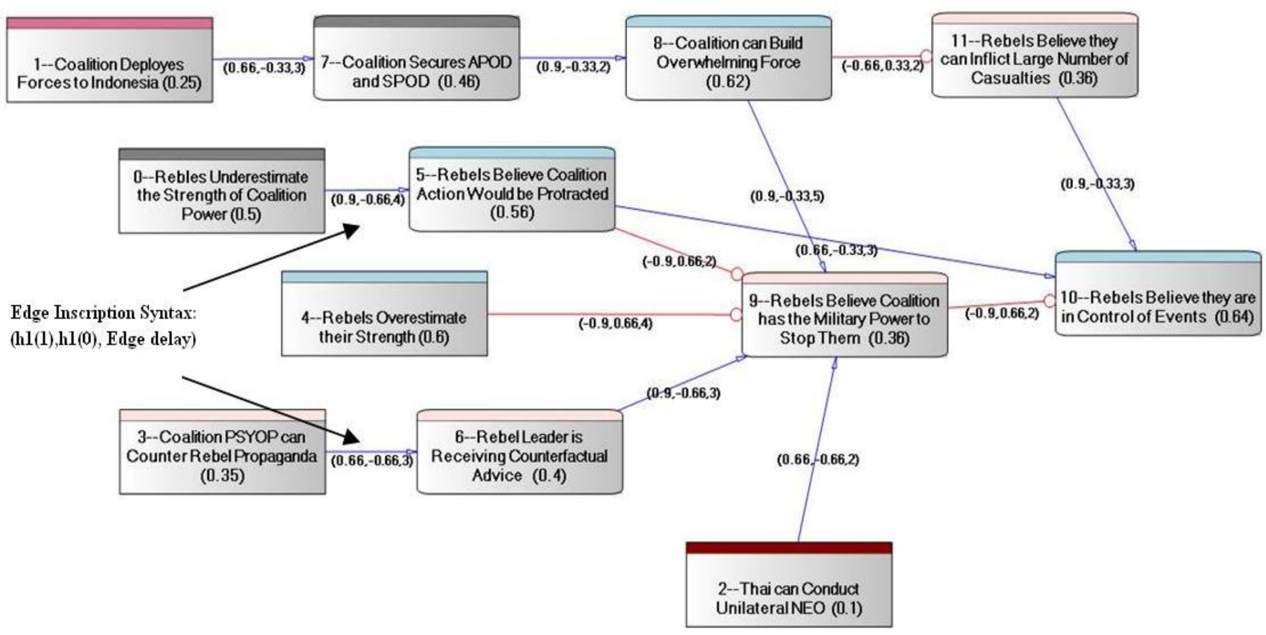


Fig. 10. Sample TIN for Analysis

Note that the simultaneous occurrence of external affecting events does not necessarily imply simultaneous revealing of their status on an affected node; the time sequence of revealed affecting events is determined by both the time stamp on each affecting event and the delays on edges. Because of the propagation delay associated with each edge, influences of actions impact the affected event progressively in time. As a result, the probability of the affected event changes as time evolves. A probability profile draws these probabilities against the corresponding time line.

The two COAs can also be described as in Table 3.

TABLE 3

Event	COA1		COA2	
	Time	Status	Time	Status
0 -- Rebels Underestimate the Strength of Coalition Power	1	1 (True)	1	1
1 -- Coalition Deploys Forces to Indonesia	1	1	2	1
2 -- Thai can Conduct Unilateral NEO	1	1	1	1
3 -- Coalition PSYOP can Counter Rebel Propaganda	1	1	2	1
4 -- Rebels Overestimate their Strength	1	1	1	1

For the same TIN model as in Fig. 10 and the corresponding course of actions, we used the approach presented in this paper and produced pertinent results for the following two cases:

Case I: For this illustration, we utilize the multiplicative influence model 1 presented in Section 2.1 and assumed the knowledge of all external affecting events. The influence constants $\{h_i(x_1^n)\}_{1 \le i \le n-1}$ are first pre-computed via the dynamic programming expression in Lemma 2. The resulting probability profiles for the two affected events/propositions in the TIN are shown in Fig. 11.

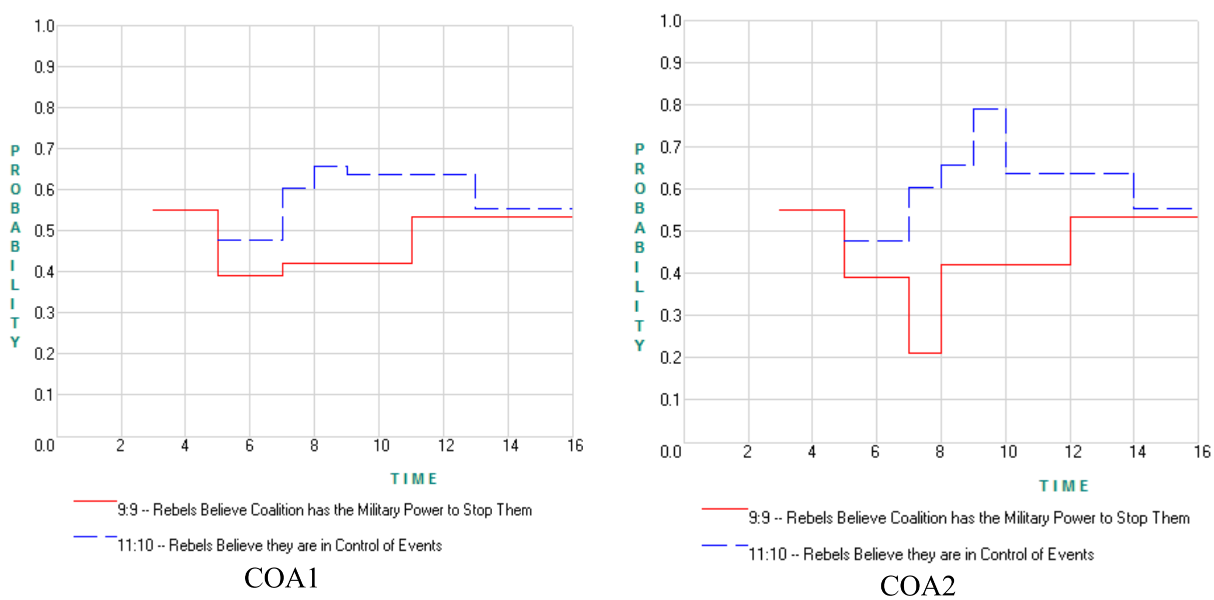
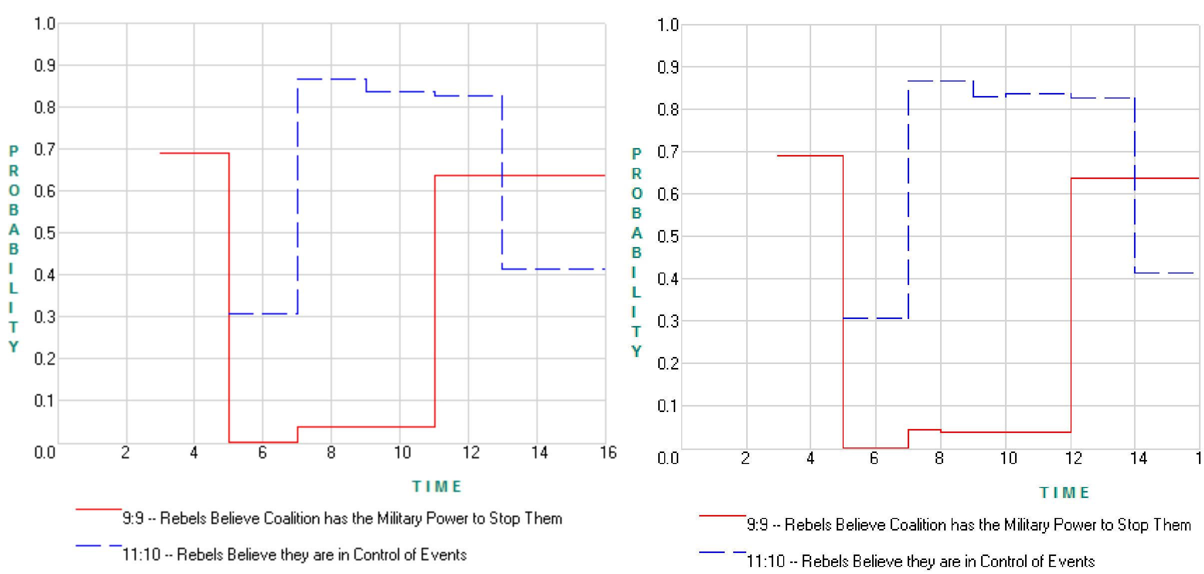


Fig. 11. Probability Profiles for Case I

4. Conclusions

Decision problems often require modeling of cause and effect relationships between disparate aspects of a domain. A typical analysis problem may require study of effects of certain events and/or actions on various aspects of a domain, where the latter are modeled as propositional statements/variables. The aspects that are affected by decisions and/or (tactical) actions may include political, military, economic, social, infrastructure, and information (PMESII) variables. The advantage of Timed Influence Nets (TIN) lies in their ability to represent, in a compact and integrated manner, both causal and time-sensitive relationships among variables that represent PMESII aspects of a domain. The TIN modeling approach is especially suitable for situations where it is difficult to either evaluate or estimate the conditional probabilities involved in the modeling; such is the case, for example, when potential human reactions and believes are being modeled.

In this paper, we studied Time Influence Networks (TIN), in the presence of either multiplicative or additive influence functions, that may be also time-varying and/or cyclic. These influence functions allow a domain expert to interleave the strengths of pair- influences in various forms. Once all the affecting events/variables are identified for an effect variable, the individual influences may be combined to represent the aggregate influence by all the affecting variables. The time-varying and cyclic functions are extensions to an earlier mapping from a TIN model to a Time Sliced Bayesian Network. The proposed time-varying influence functions allow modeling of influences whose strengths vary with time. A cyclic influence, on the other hand, provides a provision for self-promoting set of influences. The two extensions will allow for further modeling flexibility regarding the use of TINs in the representation of uncertain domains. We also studied the temporal effects induced by such functions, as exhibited by the time evolution of event-probabilities. The time evolution of event-probabilities is shown to be captured by probability profiles that map an event's probability on a timeline. These profiles can be an affective analysis tool for an analyst who, given a COA, might be interested in identifying time intervals or windows of opportunities, in terms of the likelihood of event/proposition occurrence.



COA2 COA1

Fig. 12. Probability Profiles for Case II

References

[1] Agosta J. M, —Conditional Inter-Causally independent Node Distributions, a Property of Noisy-or Models," in *Proceedings of the 7th Conference on Uncertainty in Artificial intelligence*, 1991.

[2] Drudzel M. J. and Henrion M., —Intercausal Reasoning with Uninstantiated Ancestor Nodes," in *Proceedings of the 9th Conference on Uncertainty in Artificial Intelligence*, 1993.

[3] Haider S. and Levis A. H., —An Approximation Technique for Belief Revision in Timed Influence Nets," in Proceedings of Command and Control Research and Technology Symposium, 2004.

[4] Haider S. and Zaidi A. K., —Transforming Timed Influence Nets into Time Sliced Bayesian Networks," in Proceedings of Command and Control Research and Technology Symposium, 2004.

[5] Haider S. and Levis A. H., "Dynamic Influence Nets: An Extension of Timed Influence Nets for Modeling Dynamic Uncertain Situations," in Proc. 10th International Command and Control Research and Technology Symposium, Washington DC, June 2005.

[6] Rosen J. A. and Smith W. L., —Influence Net Modeling with Causal Strengths: An Evolutionary Approach," in Proceedings of the Command and Control Research and Technology Symposium, Naval Post Graduate School, Monterey CA, June 25-28, 1996.

[7] Wagenhals L. W. and Levis A. H., "Modeling Support of Effects-Based Operations in War Games," in Proc. 2002 Command and Control Research and Technology Symposium, Monterey, CA, June 2002

[8] Wagenhals L.W., Levis A. H., and McCrabb M., "Effects Based Operations: a Historical Perspective for a Way Ahead," in Proc. 8th Int'l Command and Control Research and Technology Symposium, National Defense University, Washington, DC, June 2003.

[9] Wagenhals L. W. and Wentz L. K., "New Effects-Based Operations Models in War Games," in Proceedings of the 2003 International Command and Control Research and Technology Symposium, National Defense University, Washington, DC, June 2003.

[10] Haider S., —On Computing Marginal Probability Intervals in Probabilistic Inference Networks," in Proceedings of IEEE-SMC Conference, October, 2003.

[11] Haider S. —A Hybrid Approach for Learning Parameters of Probabilistic Networks from Incomplete Databases," Design and Application of Hybrid Intelligent Systems, Proceedings of the Third International Conference on Hybrid Intelligent Systems (HIS'03), IOS Press, Amsterdam, The Netherlands, ISBN 158603-394-8, 2003.

[12] Haider S., Zaidi A. K., and Levis A. H., —A Heuristic Approach for Best Set of Actions Determination in Influence Nets," in Proc. IEEE International Conference on Information Reuse and Integration, Las Vegas, Nov. 2004.

[13] Haider S. and Levis A. H., —Effective Courses of Action Determination to Achieve Desired Effects" in *IEEE Trans. on Systems, Man and Cybernetics, Part A: Systems and Humans*, Vol. 37, No. 6, November 2007.

[14] Wagenhals L. W., —Course of Action Development and Evaluation Using Discrete Event System Models of Influence Nets", PhD Dissertation, George Mason University, 2000.

[15] Wagenhals L. W., Levis A. H., —Course of Action Development and Evaluation," in Proceedings of the 2000 Command and Control Research and Technology Symposium, 2000.

[16] Wagenhals L. W. and Levis A. H., —Modeling Effects Based Operations in Support of War Games," in Proc. Of the 15th International Symposium on Aerospace / Defense Sensing, Internal Society for Optical Engineering, Proceedings of SPIE, Vol. # 4367, 2001.

[17] Wagenhals L. W., Shin I., and Levis A. H., —Creating Executable Models of Influence Nets with Coloured Petri Nets," Int. J. STTT, Spring-Verlag, Vol. 1998, No. 2, 1998.

[18] Wagenhals L. W., Reid T. J., Smillie R. J., & Levis A. H., —Course of Action Analysis for Coalition Operations," Proceedings of 6th International Command and Control Research and Technology Symposium, Annapolis, Maryland, Jun. 2001.

[19] Wagenhals L. W. and Levis A. H., "Modeling Support of Effects-Based Operations in War Games," in *Proc. 2002 Command and Control Research and Technology Symposium*, Monterey, CA, June 2002.

[20] Zaidi A. K., Wagenhals L W., and Haider S., —Assessment of Effects Based Operations Using Temporal Logic," in Proc. of the 10th International Command and Control Research and Technology Symposium, 2005.

[21] Haider S. and Zaidi A. K., —On Temporal Analysis of Timed Influence Nets using Point Graphs," in the Proc. of the 18th International FLAIRS Conference, FL, 2005.

[22] Zaidi A. K, Mansoor F. and Papantoni-Kazakos P., —Modeling with Influence Networks Using Influence Constants: A New Approach," 2007 *IEEE International Conference on Systems, Man, and Cybernetics, IEEE Systems, Man, and Cybernetics.* 2007.