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Where Have We Been? Where Are We Going?

An Integrated Asset Allocation and Path Planning Method to Search for Targets in a Dynamic Environment

Modeling and Simulation

Concepts, Theory, and Policy

Experimentation and Analysis

Woosun An [STUDENT]
Email: woa05001@engr.uconn.edu

Manisha Mishra [STUDENT]
Email: manisha.uconn@gmail.com

Chulwoo Park [STUDENT]
Email: chp06004@engr.uconn.edu

Krishna Pattipati*
University of Connecticut
Dept. of Electrical and Computer Engineering
371 Fairfield Way, U-2157
Storrs, CT 06269-2157, United States
FAX: 860-486-5585
Phone: 860-486-2890
Email: krishna@engr.uconn.edu

* To whom correspondence should be addressed: krishna@engr.uconn.edu

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ABSTRACT

In this paper, a moving target search problem using multiple searchers is investigated within a dynamic and uncertain mission environment. The motion of target and searchers is modeled in discrete time and space. A search region satisfying *contiguity constraint for search activities* is assigned to every searcher for a specified search interval (decision interval). At each time epoch of the decision interval, the searcher's move is constrained only to the neighboring cells within the assigned region. After the elapse of each search interval, the assets may be reassigned. A hidden Markov modeling (HMM) framework is used to formulate the asset allocation and search path selection problem as an optimization problem of maximizing the expected detection probability (equivalently, expected number of detections). In order to solve this NP-hard problem, we use a greedy approach based on the evolutionary algorithm (EA), coupled with the auction algorithm, information gain and the Voronoi tessellation approach. An ASW mission scenario involving the monitoring of an enemy submarine within a geographic region is used to evaluate our approach where we assume that the search region for each asset is recomputed periodically. We compare our results with a search path plan that does not consider contiguity constraints in search activities.

Keywords: Search problem, Search allocation, Search path, Auction algorithm, Evolutionary algorithm (EA), Voronoi tessellation, Hidden Markov models (HMM)

1. Introduction

1.1 Motivation

Search problems occur in a number of military and civilian applications, such as land mine detection and location, maritime operations (e.g., antisubmarine warfare (ASW)) and search and rescue (SAR). Generally, the search problems can be categorized as one-sided or two-sided search problems. In a *one-sided* search problem, the searcher can choose his strategy, but the target is passive in the sense that it neither has any moving strategies nor does it react to the search. Typically, search and rescue is treated as a one-sided search problem. In a two-sided search problem, on the other hand, we allow both the target and the searcher to choose their own strategies. Both stationary and moving target search problems are considered as two-sided search problems. An example of a two-sided stationary target search problem may occur when the target has strategies such as a place of choice to hide or changing movement trajectories. Most search problems involving moving targets are considered as two-sided search problems [23].

The ASW mission is a typical example of a two-sided search problem, where the attacker and the searcher are aware of the each other's presence [23]. Surface, airborne and sub-surface ASW platforms are used to search for, detect, classify, track and prosecute hostile submarines. The ISR assets on a surface platform include hull-mounted active or passive sonar systems; airborne platforms deploy sonobuoys in various patterns to enhance search effectiveness and are capable of extended echo ranging, as well as radars that can detect and localize an exposed submarine periscope or mast; and subsurface platforms feature hull-mounted or towed-array active or passive sonar systems for surveillance.

The ASW mission environment is characterized by meteorological and oceanographic (METOC) conditions which provide the commanders with the relevant information in a timely manner for necessary courses of action. Due to the dynamic nature of the environment, METOC data must be continuously collected, analyzed and disseminated in order to develop predictions. The collected data is used for assessing the impact of the current and forecasted environment on individual sensors and weapon platforms, as well as tactics in the form of performance surfaces. The commander is presented with different types of information, with different types of uncertainty associated with it, to generate plans or evaluate courses of action (COAs), before assets conduct actual ASW search activities. Combined use of air, surface, and subsurface assets enable an ASW commander to optimally position and assign ASW platforms to maximize the effectiveness of search by capitalizing on the ocean's local acoustic/surface conditions. A key challenge in ASW is the development of optimization algorithms that incorporate information from multiple sources (e.g., intelligence, environment, and sensor data) for asset selection and placement, search path planning, and sensor coordination.

Santos [16] formulated the problem of searching for a moving target using multiple searchers by maximizing the expected number of detections and applied to ASW problem. He developed heuristic and optimal approaches to solve the search problem, where the searchers can move in the entire search area, while the searcher's move is constrained only to the neighboring cells at each time epoch. However, in reality, the strategy may not be efficient, since the searchers do not consider the co-searcher's path or navigation; it might result in search conflicts, sensor interference and/or complex searcher coordination. One solution to this problem is to allocate each searcher to a specific region, while guaranteeing the contiguity of search activities, before the search activity begins. Consequently, a dynamic search plan for each asset over the assigned region can be constructed circumventing the searcher coordination issues. In addition, as the target distribution and/or motion evolve with time, the current asset assignment may not be efficient; the assets may need to be reassigned after the elapse of search interval.

In this paper, a hidden Markov modeling (HMM) framework is used to formulate a search path problem over a finite time horizon as an optimization problem of maximizing the expected detection probability (or expected number of detections) [16]. An HMM with controllable emission matrices corresponding to each asset is an appropriate way to model the search problem, because the target's motion is concealed and its true state (i.e., location) can only be inferred through the observations obtained by the assets. A pattern of these observations and its dynamic evolution over time provides the information base for inferring the target existence and its location. In order to solve the problem, we decompose the problem into two sequential phases. In phase I, we partition the entire search area into search regions subject to the contiguity constraint for search activities at the beginning of each search interval. After the partition of the search area, the assets are assigned to the search regions, depending on asset capabilities (e.g., sensing capability and sweep width). Using the output of Phase I, a dynamic search plan for each asset is constructed over the search interval. In addition to detection probability, other factors, such as risk to high-value assets, may be considered as part of the asset allocation and search path plan optimization, leading to a multi-objective optimization problem. An ASW mission scenario involving the monitoring of an enemy submarine within a geographic region is used to evaluate our approach. We compare our results with search path plan without considering contiguity constraint for search activities.

1.2 Previous Work

In modeling a target's likely motion over a time period of interest, all models assume that the prior probability distribution of the possible target locations is given (e.g., via intelligence) and the objective is to determine a search strategy to maximize the target detection probability [5]. Under the assumption that the target motion strategy does not change during the search process, its motion may simply be described by its likely locations at each successive time steps. Brown [5] proposed an optimal search strategy for a moving target in discrete space by considering the search object's motion as Markovian and the detection function to be exponential. The strategy involved repeatedly solving a stationary target search problem at each time step. Another version of Brown's algorithm was given by Washburn [27], where he generalized it to the class of Forward and Backward algorithms applied to a general class of payoff functions. Washburn [25] also investigated the efficiency of Branch-and-Bound methods for solving the target search problem. Algorithms for non-exponential detection functions and non-Markov motions may be found in [18], [19].

The search problems considered above assume that asset allocation at each time interval can be distributed over the search space regardless of any geometrical and physical constraints. However, for realistic asset allocation, we should maintain the contiguity of search space and minimize the time assets take to travel from one part of the search region to another. In these cases, we have an optimal search path problem, instead of an optimal allocation of search effort. To solve the search problems with path constraints (e.g., from the current assigned cell, the subsequent cell that an asset can search is constrained to the adjacent cells), Eagle [8] and Eagle and Yee [9] proposed a partially observed Markov decision process (POMDP) and branch-and-bound-based approach for the search problem. Even though this class of problems does not consider contiguous partitioning of a search area among multiple assets, it is still difficult to solve them; indeed, these problems are known to be *NP-hard* [21].

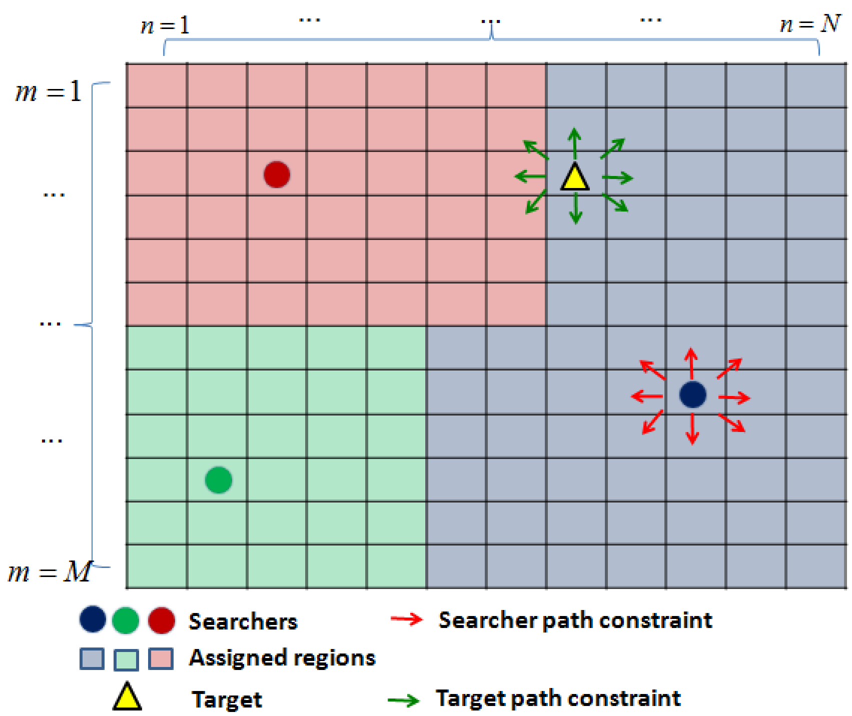

Figure. 1. Asset allocation satisfying the *contiguity constraint for search activities*

Martins [14] developed an approximation method wherein an upper bound on the search path solution can be obtained by changing the objective function to one of maximizing the expected number of detections (ED). This also simplifies the formulation, since explicit enumeration of all possible paths is not needed [14]. Hong [10] proposed a method for the single-searcher path-constrained problem by optimizing an approximation to the non-detection probability computed from the conditional probability over a fixed time window. The problem has been formulated as a shortest path problem on an acyclic layered network, where the number of layers is of the order of search duration [10].

1.3 Organization of the paper

The paper is organized as follows. In section 2, we formulate the integrated asset allocation and search path optimization problem by modeling target-asset interactions via HMM. In section 3, the search problem is solved using a greedy approach based on the EA, coupled with the auction algorithm [3] [4] and the Voronoi tessellation approach. In section 4, we apply our model and solution approach to an ASW mission scenario and present the analysis results. Finally, section 5 concludes with a summary.

2. Asset Allocation and Search Path Model

2.1 HMM Model

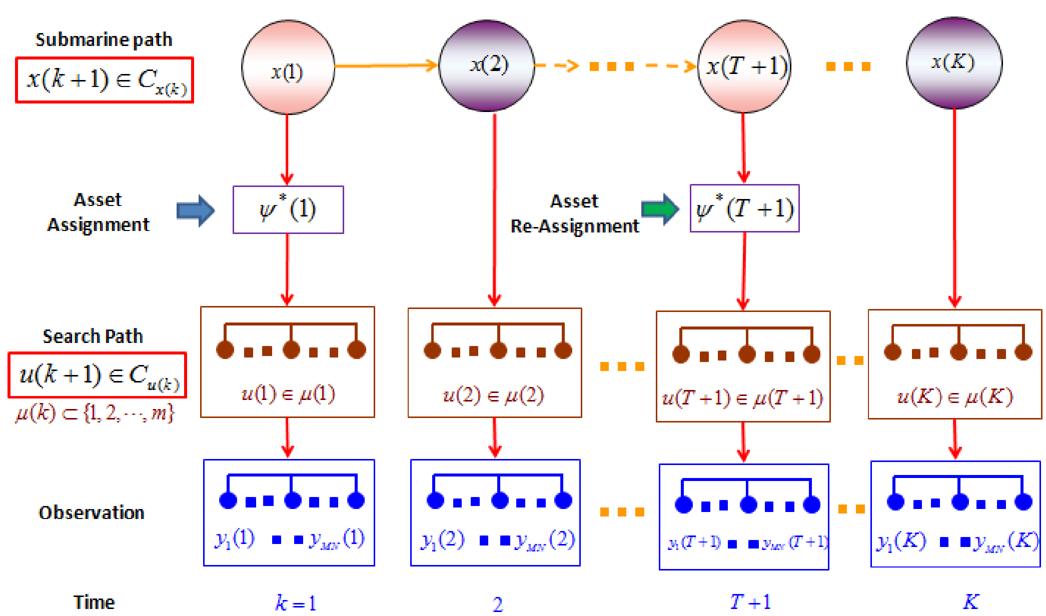

Figure. 2. Search problem using HMM

Consider a search area comprised of $N \times M$ cells. Suppose there are *m* assets available at time epoch $k = vT + 1$, $v = 0,...,(K/T) - 1$ and $\mu(k) \subseteq \{1, 2, .., m\}$ are the set of available assets that are to be assigned to regions satisfying contiguity constraint for search activities. An asset *i* assigned to a specific region moves from its current cell *j* to one of neighbor cells C_j at each time epoch $k = vT + t$, $v = 0,...,(K/T) - 1, t = 1,...T$. The target motion and asset observations are modeled using a HMM. A HMM is parameterized by a transition probability matrix $\Gamma(k)$, the set of emission matrices $\mathbf{B}(k)$, and initial probability φ . Here, we assume that the HMM parameter sets $\Lambda(k) = (\Gamma(k), \mathbf{B}(k), \varphi)$, $k = 1, 2, ..., K$, are known *a priori*. As shown in Fig.2, a target's move can be modeled by the transition probability matrix $\Gamma(k)$ of the underlying Markov chain:

$$\Gamma(k) = [\gamma_{rj}(k)] = [P(x(k) = j \in C_r \mid x(k-1) = r) \; , \; (j, r = 1, ..., NM) \tag{1}$$

where $x(k)$ is the cell location of the target at time epoch *k*. We denote the subset of *m* emission matrices, corresponding to each of the *m* assets associated to cell *j*, as $B_j(k) = \{B_{j1}(k) \cdots B_{ji}(k) \cdots B_{jm}(k)\}$. The set of emission matrices for the $M \times N$ cells is denoted by $\mathbf{B}(k) = \{B_1(k) \cdots B_j(k) \cdots B_{MN}(k)\}$. Let L denote the number of observation symbols. The symbol, measured by asset $u_j(k) = i$ assigned to cell *j* at time epoch *k*, is denoted by $y_j(k) \in \{O_{j1}(k), .., O_{jl}(k), .., O_{jL(i)}(k)\}$. Evidently, the number of observation symbols L can be a function of asset $u_j(k)$. This models a realistic scenario in which different assets have different capabilities in generating different observation symbols, depending on the assets' cell locations. If none of the assets is assigned to cell *j* at a given epoch, we assume that the observed symbol is null ($\varnothing$).

Suppose asset $u_j(k) = i$ is assigned to cell *j*. Then, the *probability* of observing symbol $O_{jl}(k)$ is related to the elements of the emission matrix, $B_{ji}(k)$ via

$$\begin{aligned} &B_{ji}(k) = [b_{jlri}(k)] = [P(y_j(k) = O_{jl}(k) \mid x(k) = r, u_j(k) = i] \\ &(j, r = 1, 2, ..., NM; l = 1, 2, ..., L(u_j(k) = i); i = 1, 2, ..., m) \end{aligned} \quad , \tag{2}$$

The initial target probability distribution of the underlying Markov states of at time $t = 0$ is denoted by

$$\varphi = [\varphi_j = p(x(0) = j)],\ (j = 1, 2, ..., NM)\,, \tag{3}$$

2.2 Information State Propagation and Update

Suppose the target is in cell *r* at time *k*. Let us define $D_j^i(k)$ as observable cells when asset *i* is in cell *j* at time *k*, and define $D(k)$ as the set of entire observable cells searched by the *m* assets at time *k*. At time epoch *k*, we have, for each cell $j = 1, ..., NM$, the information set $\{\mathbf{Y}^{k-1}, \mathbf{U}^{k-1}\}_{k=1}^{K}$ where $\{\mathbf{Y}^{k-1}, \mathbf{U}^{k-1}\} = \{Y_{h \in D_j^i(k)}{}^{k-1}, U_j{}^{k-1}\}_{j=1}^{NM}$. In cell *j*, the previously observed symbols and the asset sequence used from time epoch $t = 1$ to time epoch $t = k-1$ are defined as $Y_{h \in D_j^i(k)}{}^{k-1} = \{y_{h \in D_j^i(1)}(1), ..., y_{h \in D_j^i(k-1)}(k-1)\}$ and $U_j{}^{k-1} = \{u_j(0), u_j(1), ..., u_j(k-1)\}$. Given the available information, $\{\mathbf{Y}^{k-1}, \mathbf{U}^{k-1}\}$, the information state vector $\underline{\pi}(k \mid k-1)$ is the sufficient statistic to infer the target location at time epoch *k*. Indeed, the information state is the *predicted* probability, given by

$$\begin{aligned}
\underline{\pi}(k \mid k-1) &= \{\pi_1(k \mid k-1), \cdots, \pi_{MN}(k \mid k-1)\}^T \\
&= \{P(x(k) = 1 \mid \mathbf{Y}^{k-1}, \mathbf{U}^{k-1}), \cdots, P(x(k) = MN \mid \mathbf{Y}^{k-1}, \mathbf{U}^{k-1})\}^T \\
&= \Gamma^T(k)\underline{\pi}(k-1 \mid k-1)
\end{aligned} \tag{4}$$

Then, using observation probability, we obtain the updated information state $\underline{\pi}(k \mid k)$ by using the forward algorithm;

$$\begin{aligned}
\pi_h(k \mid k) &= \frac{(1 - b_{jlri}^h(k))\pi_h(k \mid k-1)}{\sum_{i=1}^{m}\left(\sum_{q \in D_f^i(k),\, f \in A_i}(1 - b_{flri}^q(k))\pi_q(k \mid k-1) + \sum_{v \notin D_f^i(k),\, f \in A_i}\pi_v(k \mid k-1)\right)} \\
&= \frac{(1 - b_{jlri}^h(k))\pi_h(k \mid k-1)}{1 - \sum_{i=1}^{m}\sum_{q \in D_f^i(k),\, f \in A_i} b_{flri}^q(k)\pi_q(k \mid k-1)},\ h \in D(k)
\end{aligned} \tag{5}$$

$$\pi_h(k \mid k) = \frac{\pi_h(k \mid k-1)}{1 - \sum_{i=1}^{m}\sum_{q \in D_f^i(k),\, f \in A_i} b_{flri}^q(k)\pi_q(k \mid k-1)},\ h \notin D(k) \tag{6}$$

Note that $\underline{\pi}(0 \mid -1) = \varphi$.

3. Search Region Partition and Search Path Problem Formulation and solution approach

3.1 Problem Formulation

Given total search time of K time units, we want to define the allocation area $A_i(vT+1),\ i=1,...,m,\ v=0,...,(K/T)$ of each asset and the search path of each asset $\psi_{ij}(k),\ i=1,...,m,\ j=1,...,NM$ during search interval $k=vT+t,\ t=1,...,T$, maximizing the detection probability of target. Here, T is the search interval, at the end of which search area may be re-partitioned among the assets, and assume, for simplicity that K is divisible by T[1]. Martins [14] developed an approximation method wherein an upper bound of the search path solution can be obtained by changing the objective function to the problem of maximizing the expected number of detections (ED). This also simplifies the formulation, since explicit enumeration of all possible paths is not needed [14]. Then, using ED, we can formulate a search path problem as follows:

$$\max_{\psi} \sum_{v=0}^{(K/T)-1} \sum_{t=1}^{T} \sum_{i=1}^{m} \sum_{j\in A_i(Tv+1)} \sum_{h\in D_j^i} \pi_h(k\mid k-1) b_{jlri}^h(k) \psi_{ij}(k)$$

where $k=Tv+t, v=0,...,(K/T)-1,\ t=1,...,T$

$$\text{s.t.} \sum_{i=1}^{m} A_i(Tv+1)=A, A_i\cap A_j=\varnothing, i\neq j, v=0,...,(K/T)-1 \quad \text{(a)}$$

$$\sum_{j\in A_i(Tv+1)} \psi_{ij}(vT+1)=1,\ \forall i,\ v=0,...,(K/T)-1, \quad \text{(b)} \quad (7)$$

$$\sum_{j\in A_i(Tv+1)} \psi_{ij}(vT+t) - \sum_{C_j\in A_i(Tv+1)} \psi_{iC_j}(vT+t+1)=0,\ \forall i,\ t=2,...,T-1,\ v=0,...,(K/T)-1, \quad \text{(c)}$$

$$\sum_{C_j\in A_i(Tv+1)} \psi_{ij}(vT)=1,\ \forall i,\ v=1,...,(K/T)-1 \quad \text{(d)}$$

Asset i should travel to all regions of A_i without crossing over $A_n, i\neq n$ (e)

$\psi_{ij}(k)\in\{0,1\},\ i=1,...,m,\ j=1,...,MN, k=1,...,K$ (f)

[1] When $T=1$, we have a truly dynamic search area partitioning and search path optimization. When $T=K$, we have static search area partitioning and dynamic search path optimization. When $1<T<$ K, we have a semi-dynamic search area partitioning and dynamic search path optimization.

The constraint (a) ensure that the assets are assigned to mutually exclusive areas (i.e., no two assets are assigned to the same region). The constraint in (e) implies the contiguity constraint for search activities. The constraints (b), (c), and (d) are used to ensure that the searcher's move is constrained only to the neighboring cells within the assigned region (i.e., if asset is in the cell *j*, next move should be constrained to neighboring cells C_j).

In order to solve the problem, we use a greedy approach of decomposing the problem into two sequential phases. In phase I, we partition the search area into search regions at the beginning of each search interval, subject to contiguity constraint for search activities. Using the output of Phase I, a search plan is created for each asset over the search interval. In next section, we describe the approach in detail.

3.2 Phase I: Asset allocation (*Search Region Partition*)

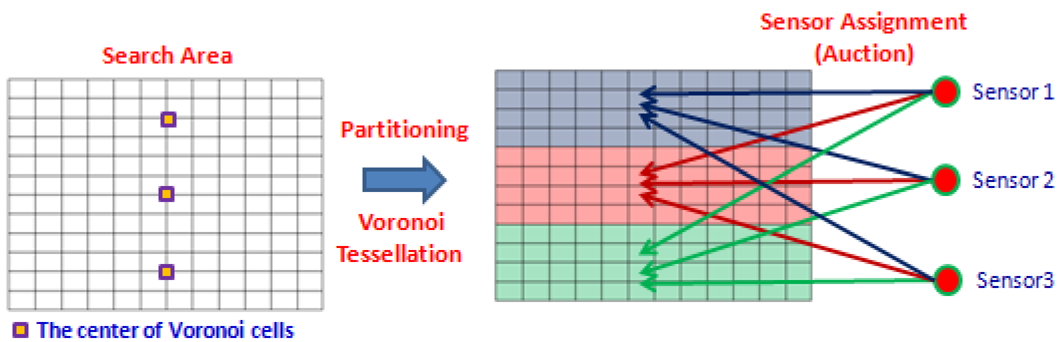

Figure.3. Illustrative Example of Phase 1

Phase I employs the EA [6] coupled with the Voronoi tessellation approach for partitioning and the auction algorithm for the assignment problem. The auction algorithm, proposed by Bertsekas *et al.* [3], [4], is the most efficient algorithm for solving the (2-D) assignment problems, where it consists of a bidding phase and an assignment phase and an optimal assignment is found by employing a coordinate descent method on the dual function. Here, Voronoi tessellation is used for ensuring contiguity and to obtain a near-optimal solution to an otherwise intractable problem. The Voronoi (or Dirichlet) tessellation is a partition of the search space into cells such that each of the cells consists of area closer to one particular center of the cell (also called Voronoi site) than to any other site based on some metric. Typically, the Euclidean distance metric is used.

In *EA*, each individual in the population corresponds to the set of centers of Voronoi cells. Voronoi tessellation for each individual is generated at each epoch by calculating the distance between the centers of the search cells $j=1,\ldots,MN$ and the centers of Voronoi cells, as shown in Fig.3. Each cell $j=1,\ldots,MN$ is assigned to the closest center of Voronoi cells. Given the partitioned search area $A_i(Tv+1),\ i=1,\ldots,m$, at time epoch $k=Tv+1$, the asset allocation problem can be written as:

$$\begin{aligned}
&\max_{\zeta}\sum_{i=1}^{m}\sum_{j\in A_i(Tv+1)}\sum_{h\in D_j^i}\pi_h(k\,|\,k-1)b_{jlri}^{h}(k)\zeta_{ij}(k)\\
&\text{where } k=Tv+1, v=0,\ldots,(K/T)-1\\
&\text{s.t. } \sum_{i=1}^{m}\zeta_{ij}(k)=1,\ \forall j,\ v=0,\ldots,(K/T)-1 && \text{(a)} \quad (8)\\
&\qquad \sum_{j=1}^{MN}\zeta_{ij}(k)=|A_i(Tv+1)|,\ \forall i,\ v=0,\ldots,(K/T)-1 && \text{(b)}\\
&\qquad \zeta_{ij}(k)\in\{0,1\},\ \forall i,\forall j,\ v=0,\ldots,(K/T)-1 && \text{(c)}
\end{aligned}$$

The problem in (8) involves two-dimensional (2-D) assignment or a weighted bipartite matching problem, where one set of nodes corresponds to assets $i=1,\ldots,m$ and the other set to partitioned search area $A_i(Tv+1),\ i=1,\ldots,m$. When allocating m assets among m partitioned search areas at each time epoch $k=Tv+1$, one needs to consider the m x m gain matrix, which is generated for each asset - partitioned search area pair by evaluating the objective function in (8) for each asset i. The auction algorithm is used here to obtain an assignment for maximizing the objective function in (8). A simple illustrative example is shown in Fig.3. Thus, each individual's fitness is the objective function value associated with the assignment problem of allocating assets to Voronoi tessellation. The individuals at the next iteration are generated based on current individual's fitness, and this process continues until the allocation solution converges. Note that the information state $\underline{\pi}(k\,|\,k-1)$ is not updated during asset allocation in (8).

3.3 Phase II: Search Path

Considering the search path constraint (e.g., from the current assigned cell, the subsequent region that an asset can search is constrained by adjacent cells), we can formulate the search path optimization for search interval $k = Tv + t$, $t = 1,...,T$ as follows:

$$\max_{\psi} \sum_{i=1}^{m} \sum_{t=1}^{T} \sum_{j \in A_{i(Tv+1)}} \sum_{h \in D_j^i} \pi_h(k \mid k-1) b_{jlri}^h(k) \psi_{ij}(k)$$

where $k = Tv + t,\ t = 1,...,T$

$$\text{s.t.} \sum_{j \in A_{i(Tv+1)}} \psi_{ij}(vT+1) = 1, \qquad \text{(a)}$$

$$\sum_{j \in A_{i(Tv+1)}} \psi_{ij}(vT+t) - \sum_{C_j \in A_{i(Tv+1)}} \psi_{iC_j}(vT+t+1) = 0,\ \forall i,\ t = 1,...,T-1, \qquad \text{(b)} \tag{9}$$

$$\sum_{C_j \in A_{i(Tv+1)}} \psi_{ij}(v(T+1)) = 1,\ \forall i, \qquad \text{(c)}$$

$$\psi_{ij}(k) \in \{0,1\},\ i = 1,...,m,\ j = 1,...,MN, k = 1,...,K$$

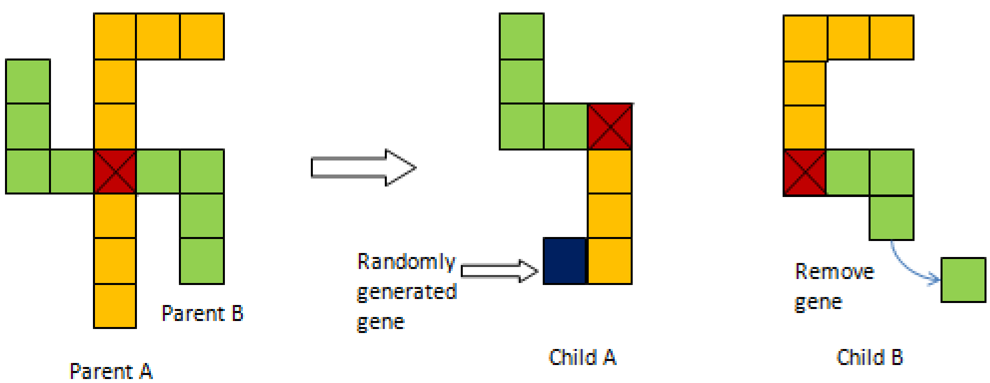

Figure.4. Crossover process for next generation of search path

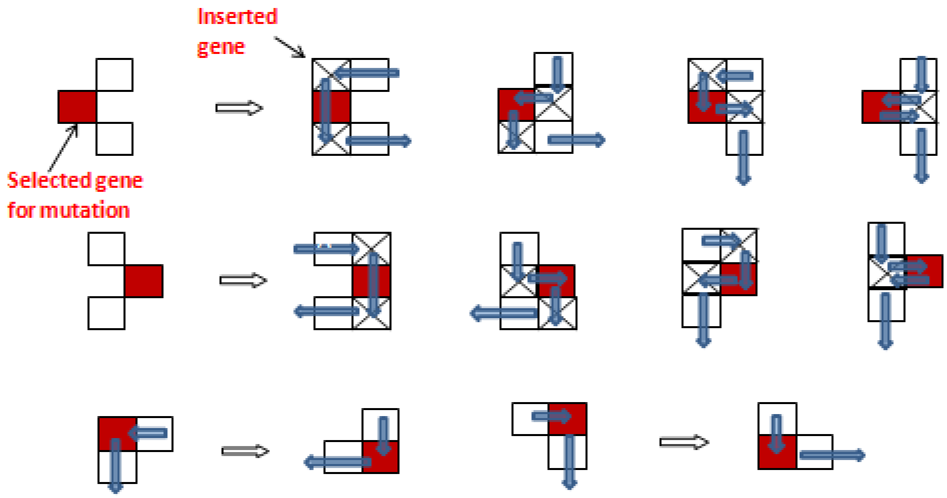

Figure.5. Mutation process for next generation of search path

We solve the optimal search problem via EA. In EA, each individual in the population corresponds to a search path. The individuals undergo crossover and mutation processes to reproduce individuals for the next iteration. If parents *A* and *B* are selected for crossover and if they share any common genes, the child can be produced by exchanging the segments of parent A with that of parent *B* as shown in Fig.4. For mutation, a gene is randomly selected and mutated by displacing it from the original location (e.g., left, right, up or down), and additional genes are introduced to maintain a contiguous search path. In addition, some genes are removed to satisfy the total search effort constraint. The process is shown in Fig.5. The fitness of the new individuals is evaluated by using the objective function in (9) for the corresponding asset. The information state $\underline{\pi}(k \mid k-1)$ is updated according to best individual's search sequence generated during time epoch $k = vT + t$, $t = 1,...,T$.

4. Computational Results

We consider an ASW mission scenario as an application of the proposed approach. The scenario involves monitoring an enemy submarine within a geographic region. We assume that searchers are provided with an initial target distribution (e.g., initial information state) and target motion profile (e.g., transition matrix). The search assets detect a target submarine by using active sonar.

4.1 Modeling the Emission Matrix (*Observation Probability*)

Active acoustic systems generate sound using an underwater projector. These sound waves propagate through the ocean to the target, reflect or scatter from the target's hull, and then propagate through the ocean to the receiver [22]. The active sonar equation is given by (asset index *i*, asset location *j*, target location is *r*, observation from cell *h*, are omitted for clarity):

$$SE = (SL - TL_{ST} - TL_{TR} + TS) - (NL - DI) - DT, \tag{10}$$

Here, *SL* (Source level) represents the amount of sound radiated by the sonar's own projector, TL_{ST} indicates the transmission loss from source to target, TL_{TR} indicates the transmission loss from target to receiver. The target strength (*TS*) is the ability of the target to reflect, or scatter, which represents the energy back to the receiver. The level of noise in the surrounding sea is denoted by *NL*, the directivity index (*DI*) is the array gain of the receiver sonar system, and *DT* is the detection threshold of the receiver sonar system.

The observation probability is modeled by incorporating the active sonar equation to detect a transiting enemy submarine. Suppose the position of the enemy submarine is in cell *r* at time *k*. Let us define $D_j^i(k)$ as observable cells when asset *i* is in cell *j* at time *k* and define $D_j^i(k),\ \ j \in A_i, i = 1,\dots,m$ as the set of observable cells of asset *i* at time *k*. Then observation probability of asset *i* assigned to cell *j* is given by:

$$\begin{aligned}
&P(y_r(k)=1 \mid x(k)=r, u_j(k)=i) = q_{rji}^r = \Phi(SE_{rji}^r / \sigma(A_i)); r \in D_j^i(k) && (a)\\
&P(y_r(k)=0 \mid x(k)=r, u_j(k)=i) = 1 - q_{rji}^r\ ;\ r \in D_j^i(k) && (b)\\
&P(y_h(k)=0 \mid x(k)=r, u_j(k)=i) = q_{hji}^r = 1 - \Phi(SE_{hji}^r / \sigma(A_i)); h \in D_j^i(k), r \in D_j^i(k),\ r \neq h && (c)\\
&P(y_h(k)=1 \mid x(k)=r, u_j(k)=i) = 1 - q_{hji}^r; h \in D_j^i(k), r \in D_j^i(k),\ r \neq h && (d)\\
&\text{if } (h \notin D_j^i(k), u_j(k)=i) \ \Rightarrow\ \text{null observation } (\varnothing) && (e)
\end{aligned} \tag{11}$$

where signal excess (*SE*) is assumed to be a normal random variable with mean 0 and standard deviation $\sigma(A_i)$, and Φ is the cumulative normal distribution [11]. $y(k)=1$ represents detection and $y(k)=0$ represents non-detection. In the context of detection theory, equation (a) is detection probability, equation (b) is miss probability, equation (c) represents non-existence target probability, and equation (d) is false alarm probability. The standard deviation $\sigma(A_i)$ is modeled as:

$$\sigma(A_i) = b + \beta \times \frac{A_i}{A = \bigcup_{j=1}^{m} A_j}, \; i = 1,...m, \; i \neq j, \; A_i \cap A_j = \varnothing \tag{12}$$

where b and β are used as scale factors. Note that $\sigma(A_i)$ increases as the area assigned to an asset i is increased. Typically, $\sigma(A_i)$ is 3-9 dB.

4.2 Interference Model for active sonar

Suppose assets i and w are conducting the search activity in non-overlapping areas A_i and A_w respectively, where asset i and w are in cells j and l at time k. Then, the interference region is defined as the intersection of observable cells $(D_j^i(k) \cap D_l^w(k)) \neq \varnothing$, as shown in Fig.6. We assume that assets do not observe any valuable information within the interference region (e.g., $y(k) =$ null observation $(\varnothing)$).

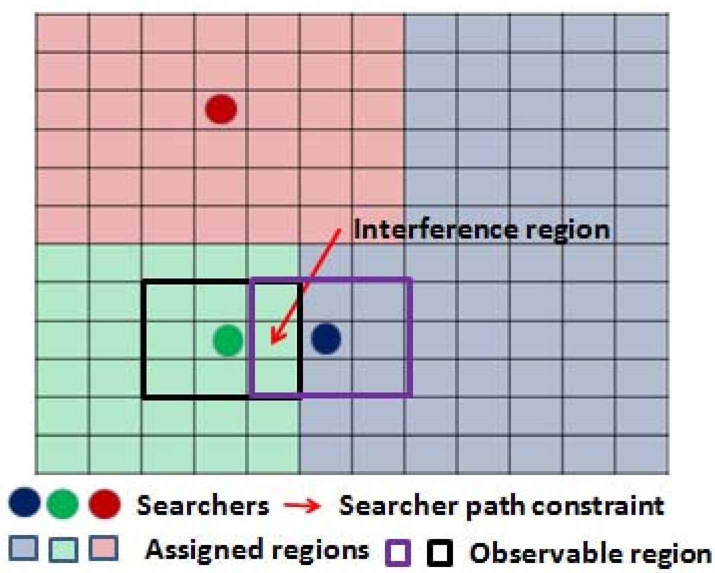

Figure.6. Interference model

4.3 Example 1: A Single Searcher and A Single Target

Here, we consider a single searcher problem modeled by Eagle [8], with different path constraints, where the searcher can only observe a cell $D_j(k) = j$ occupied at time epoch *k*. The objective here is to maximize the detection probability. This single searcher problem can be considered as a simplified version of our integrated asset allocation and path planning approach, since one asset is allocated to the entire search area which need not be partitioned. It means that only Phase II is used to obtain a search path in single searcher problem.

Eagle [8] formulated the problem as a finite horizon POMDP and employed dynamic programming recursion to find the optimal search path. For detailed problem formulation, approach, and computational times needed to solve the problem, the reader is referred to [8]. In Eagle's model, the searcher's move is constrained to 5 move options (i.e., no move, up, down, left, right), while we assume that the searcher's move is constrained only to 4 neighboring cells (i.e., up, down, left, right), that is, we have one less option in our problem. The total search time allotted to each asset is K=10 time units, and the search asset has perfect observation capability, that is, if the asset and target occupy the same cell, the target is detected. The target remains in the previously occupied cell with probability 0.4 and moves to an adjacent cell with probability 0.6/(the number of adjacent cells). Table I shows the results. Note that when the searcher initiates search in cell 1, the optimal detection probabilities are the same as in [8], which implies that the optimal path does not include 'no move' option. However, if the search is initiated in other cells, the detection probability differs in both cases because of the fact that the searcher has the freedom to wait in any cell in the Eagle's model, whereas we do not allow it in our model; this results in a slightly lower detection probability in our case. Results are obtained on a 2.20GHZ CPU, and 2GB RAM memory PC. Population size of GA is set as 20, and the number of generations is set as 100. We measure the computation time by averaging over 50 Monte Carlo runs and the calculations required approximately 12 seconds for each starting cell.

Starting Cell	Optimal Search paths (5 next move options)	Detection Probability	Optimal Search paths (4 next move options)	Detection Probability
1	2 5 6 9 8 5 6 5 8 5	0.7786	2 5 6 9 8 5 6 5 8 5	0.7786
	4 7 8 9 6 5 8 5 6 5	0.7786	4 7 8 9 6 5 8 5 6 5	0.7786
2	3 6 9 8 8 5 6 5 8 5	0.8631	3 6 9 8 5 6 5 8 5 4	0.8615
	5 6 9 8 8 5 6 5 8 5	0.8631	5 6 9 8 5 6 5 8 5 4	0.8615
3	6 6 9 8 8 5 6 5 8 5	0.8631	6 9 8 5 6 5 8 5 6 5	0.8476
4	7 8 9 6 6 5 8 5 6 5	0.8631	7 8 9 6 5 8 5 6 5 2	0.8615
5 , 7	8 8 9 6 6 5 8 5 6 5	0.8631	8 9 8 5 6 5 8 5 6 5	0.8476
6, 8, 9	9	1	9	1

Table.I. Detection Probabilities for 5 and 4 next move options

4.4 Example 2: Detection probability as a Function of Emission Probability

The sea state (e.g., wave height, period, power spectrum) impacts sensor capabilities (i.e., emission matrix). In this section, we show how our model can be used to analyze the effects of changes in the observation (emission) probabilities on the probability of detection[2]. Here, we reconsider the single searcher problem modeled by Eagle [8], with 4 move options (i.e., up, down, left, right). For illustrative purposes, we model the observation probability of asset i assigned to cell j as $q_{ji} = \exp(-\alpha_a)$ where $\alpha_a \geq 0$ is the environmental factor. Fig.7 shows the detection probability corresponding to the changing environmental factor $0 \leq \alpha_a \leq 1$, where the starting cells of the searcher are cell 1 and 2, respectively

[2] Our model also facilitates the analysis of the effects of changes in target transition dynamics.

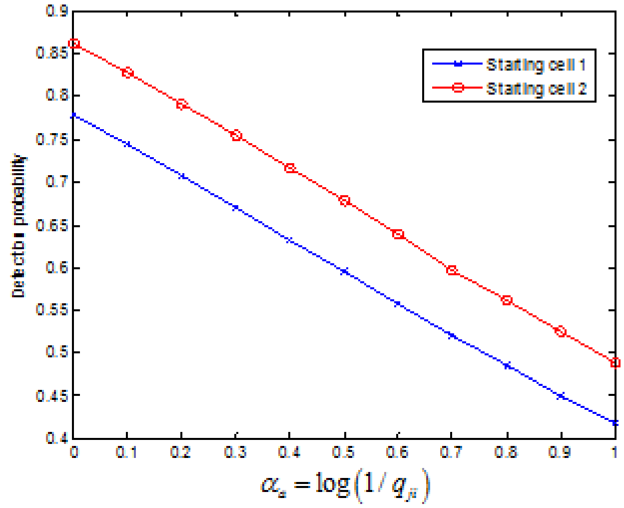

Figure.7. Change in target detection probability with $\alpha_a = \log(1/q_{ji})$

When $\alpha_a = 0$, the scenario is exactly the same as in the previous example. As α_a increases from 0 to 1, the detection probability decreases almost linearly with α_a. However, note that α_a is related to the logarithm of the inverse of observation probability via $\alpha_a = \log(1/q_{ji})$. Thus, the detection probability is proportional to log(q_{ij}) in this case. Although not discussed here, our model accommodates time varying transition and emission matrices.

4.5 Example 3: Multiple Searcher Scenario

For multiple searcher mission scenario, we assume a search area comprised of 20x20 cells and we have 3 equally capable search assets (e.g., 9 cells are observable with equal observation probability). The *SE* for detection probability (non-existence target probability) is set to 4.3 (-4.3) and 3.4 (-3.4) for an occupied cell and the eight neighboring cells respectively. In (12), b is set to 2.5 and β is selected as 6. The detection probability P_d is used as the only objective function to be maximized. The probability for the initial target position $\underline{\pi}(0\,|\,0)$ is set as uniform over the entire search area. The total search time allotted to each asset is K=40 time units, and the assets may be reallocated at time k=20, *i.e.*, at T =20. We assume that the searcher's move is constrained only to the neighboring cells (i.e., up, down, left, right) and the target transition probabilities among neighboring cells are distributed uniformly. Then, the target transition probability matrix Γ can be modeled as a 400x400 matrix as shown in Fig.8. We set the population size of EA as 20 and the number of generations (iterations) as 400. Fig.9 shows first asset allocation and information state $\underline{\pi}(1|1)$ at time k=1. Fig.10 shows the information state $\underline{\pi}(10|10)$ and $\underline{\pi}(20|20)$. After asset reallocation at k=20, the information state $\underline{\pi}(30|30)$ and $\underline{\pi}(40|40)$ are shown in Fig.11. Intuitively, search path of the asset is to track highly uncertain cells (e.g., red cells in Figs. 10 and 11). The search paths of assets obtained by using our algorithm are commensurate with our intuition.

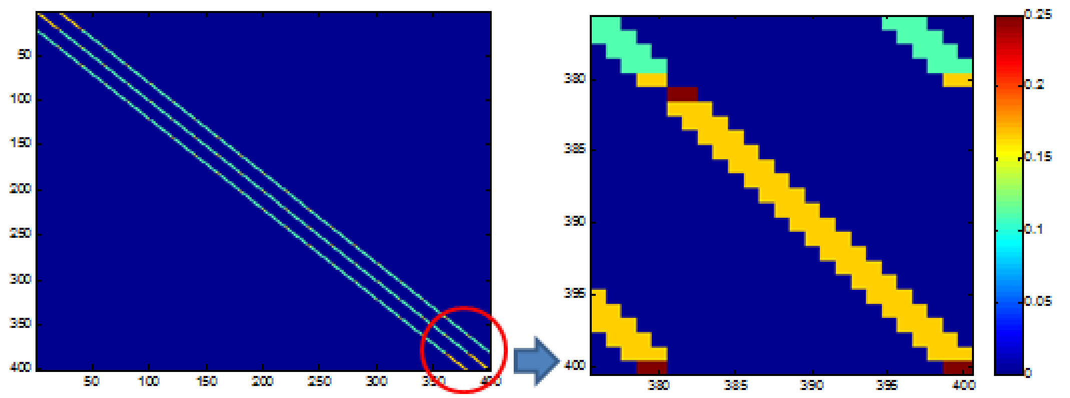

Figure.8. Search path test based on probability map

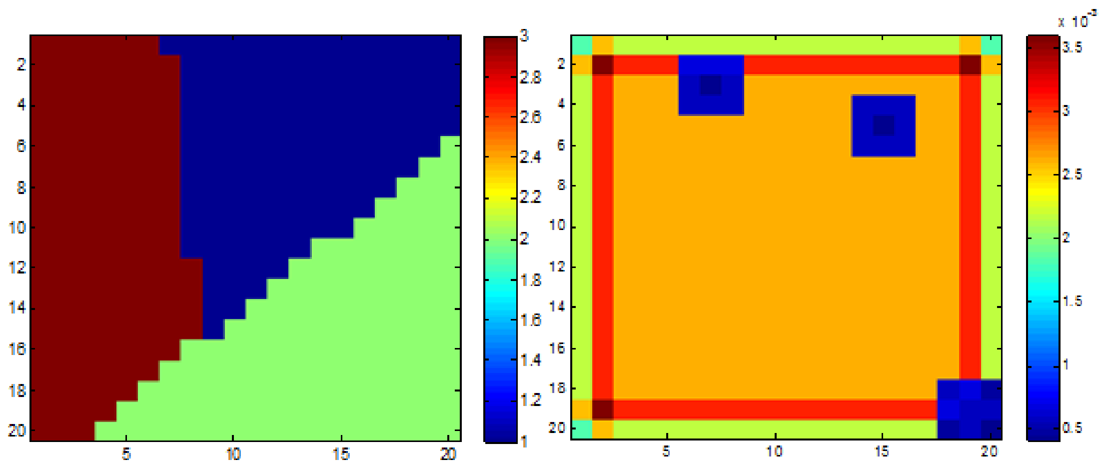

Figure.9. Assignment at *k*=0 and information state at time *k*=1

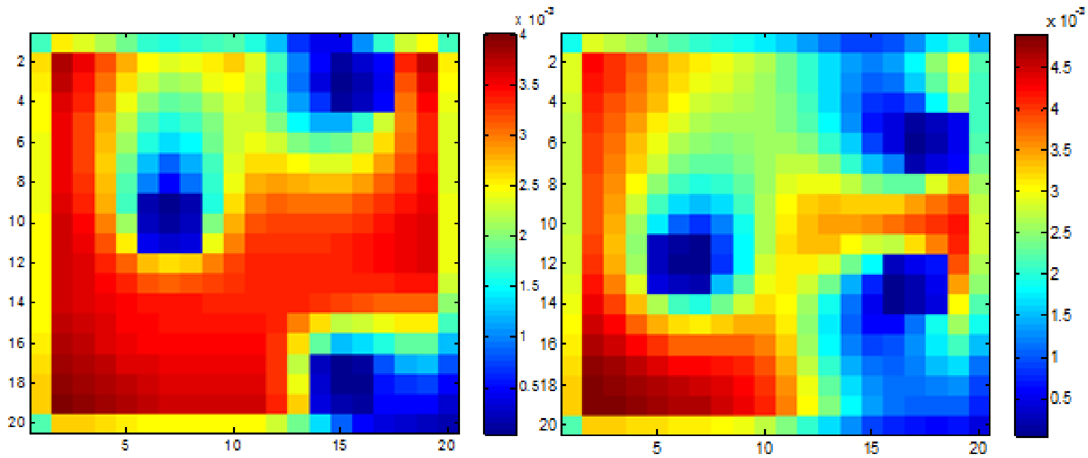

Figure.10. Information state at time *k*=10 and *k*=20

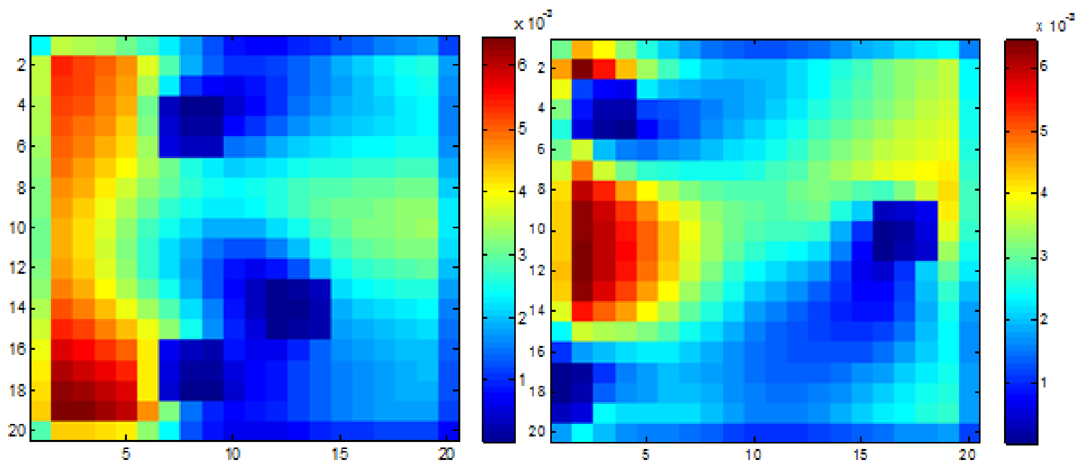

Figure.11. Information state at time *k*=30 and *k*=40

4.6 Comparison of search path planning with and without pre-assigned search regions (Phase I) for assets

Here, we compare search path planning for assets with and without pres-assigned search regions generated by using Phase I. For a valid comparison, we maintain the same search effort as the search area is varied. For this, we change the search time for the same number of assets (or number of assets for the same search time) so that the coverage rate defined by

$$\text{Coverage rate} = \frac{K \times m}{\text{Map size}} \tag{13}$$

is the same. Here m is the number of assets and K is the number of search time steps. Here, the coverage rate is set as 0.6 and the search activity is conducted by 4 assets having equal search capabilities (e.g., 9 cells observable, equal observation probability). The search time K is determined by using (13), as map size changes from 10 x10 to 16 x 16. Other parameters (e.g., *SE, b*, and β) are the same as used in the previous example. Population size of GA is set as 20, and the number of generations is set as 2000. Note that we do not consider the traveling cost due to the asset reallocation, in this experiment.

Fig.12 shows the scaled gain, by normalizing the cost function in (9) without pre-assigned asset allocation regions relative to one with pre-partitioned search regions. The search path planning with pre-assigned search region for each asset has approximately 10~20 % higher gain as compared to one using search path planning without pre-assigned asset search regions. Given a limit on the population size and number of generations, the computational complexity for each asset is significantly less when the search area is partitioned and the total search time of K time units also breaks into $(K/T)-1$ search intervals. This is because the computational complexity grows quadratically with the number of cells and linearly with the search time.

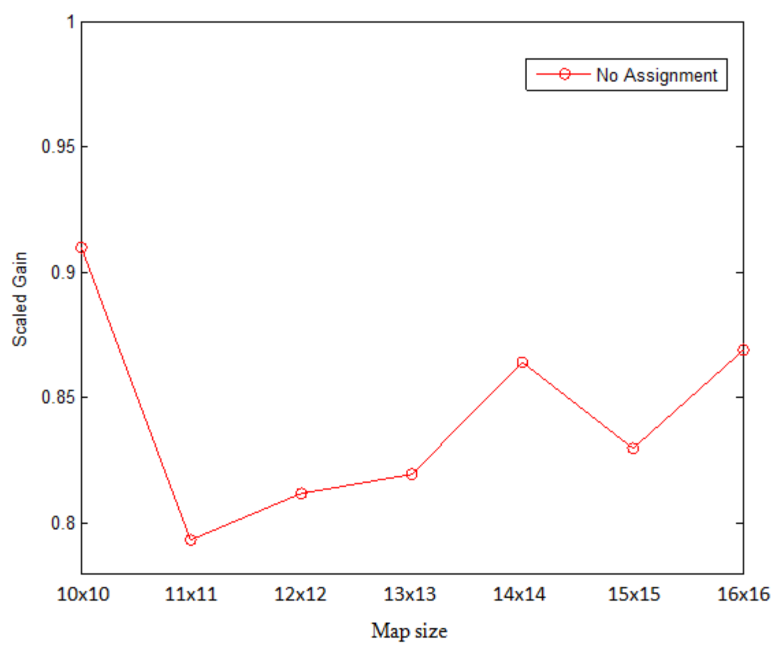

Figure.12. Comparison of search path planning with and without pre-assigned asset allocation area

5. Conclusion

In this paper, a hidden Markov modeling (HMM) framework is used to formulate search path problem of maximizing the expected detection probability. We propose a search strategy satisfying contiguity constraint for each asset's search activities. This strategy uses a greedy approach based on the evolutionary algorithm (EA), coupled with the auction algorithm, information gain and the Voronoi tessellation approach. Our approach is simulated using an ASW mission scenario of monitoring an enemy submarine in a given search area. We quantify the value of partitioning the search area among assets by compare our results with a search plan without considering the contiguity constraints for search activities. Although, our pre-assigned search path approach does not guarantee an optimal solution, it has the ease of finding a near-optimal solution with significantly less computational effort for an intractable optimization problem.

We plan to develop extensions to HMM-based search path planning model to handle the strategic interactions among multiple searchers and targets, using game theory. An advantage of using game theoretic approach for handling intelligent adversary is the rationality assumption that the gain/loss to the searcher or target is through payoffs. We plan to pursue these extensions in the future.

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