An Integrated Asset Allocation and Path Planning Method to Search for a Moving Target in a Dynamic Environment

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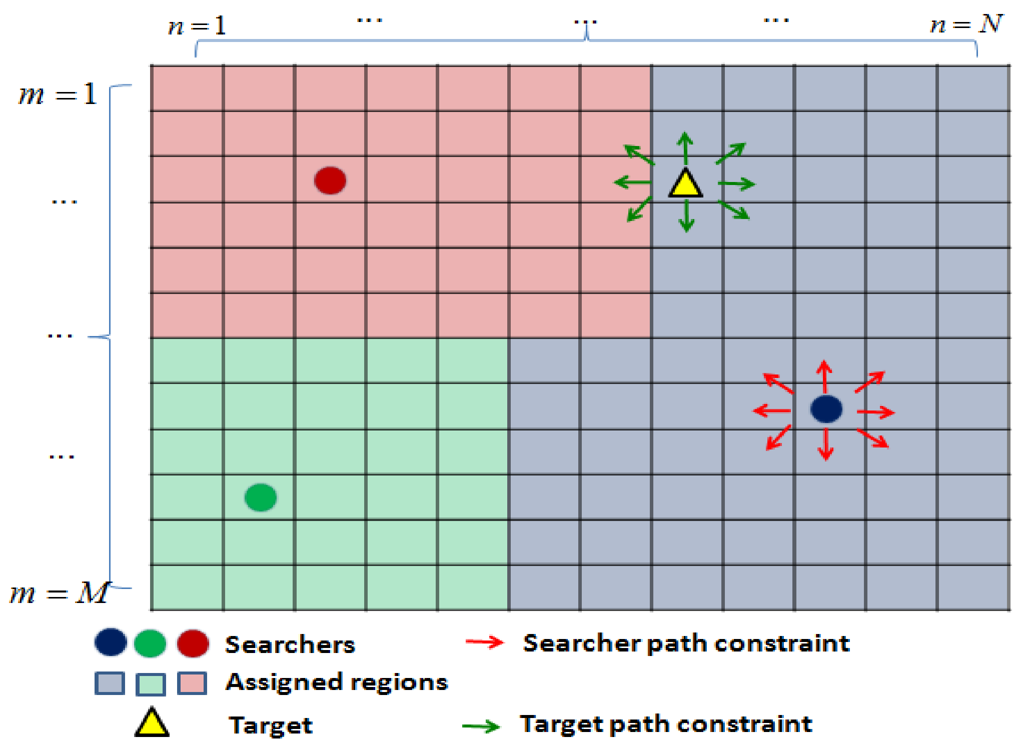
Outline

- Anti-submarine Warfare (ASW) Search Problem
  - ASW search issues
- Search Planning using Hidden Markov Models (HMM)
  - Asset/sensor capability modeled using emission matrix
  - Submarine trajectory modeled using transition matrix
- Search Planning Involves Two Phases
  - Phase 1 : Search region partition and asset allocation
    - Geographic partitioning via an evolutionary algorithm, coupled with a Voronoi tessellation approach
    - Assignment of regions via Auction algorithm
  - Phase 2 : Search path optimization
    - Optimal search path via an evolutionary algorithm
- Simulation Results
- Summary

ASW Allocation and Search Issues

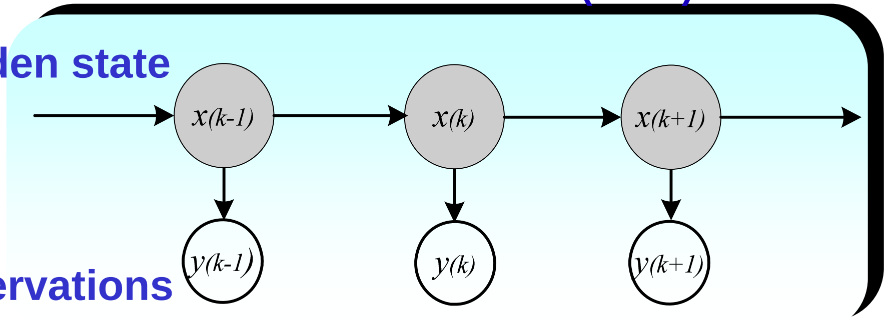


ASW search issues

- Search should be contiguous
- Conflict avoidance among search assets
- Dynamic asset assignment and search paths
- Meteorological and Oceanographic effects on search assets and targets
- Multiple objectives (e.g., maximize probability of detection, minimize risk to high value assets (e.g., carrier), etc.)

HMM-based Asset Allocation / Search Planning - 1

Hidden Markov Model (HMM)



MMs Describe Dynamics of Uncertain Tasks

- HMMs are useful for analyzing sequential data, where the true states of a task are hidden (e.g., enemy submarine's location)
- Enables us to infer the true task state from uncertain observations (including intelligence)

MM Model Parameters

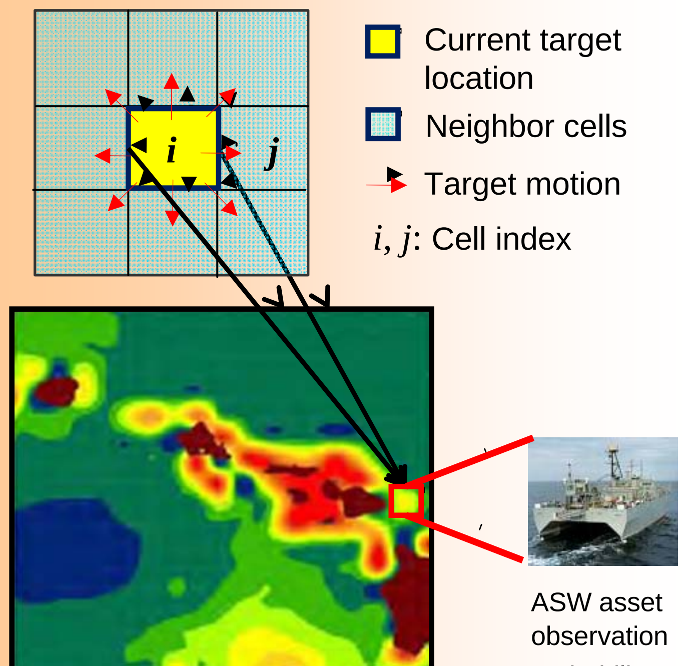
- Prior probability vector (Intelligence)
- State transition dynamics (Transition matrix), e.g., environment, submarine motion patterns
- Uncertain observations (Emission matrix), e.g., asset/sensor capability to detect targets
  - Imprecise measurements of state
  - Different information sets and accuracies

Prior probability vector

$$\underline{\pi} = [\pi_i] = P(x(1) = S_i)$$

Transition matrix

$$A = \left[a_{ij}\right] = \left[P(x(k+1) = j \mid x(k) = i)\right]$$



Pd (emission probability) map

Emission matrix

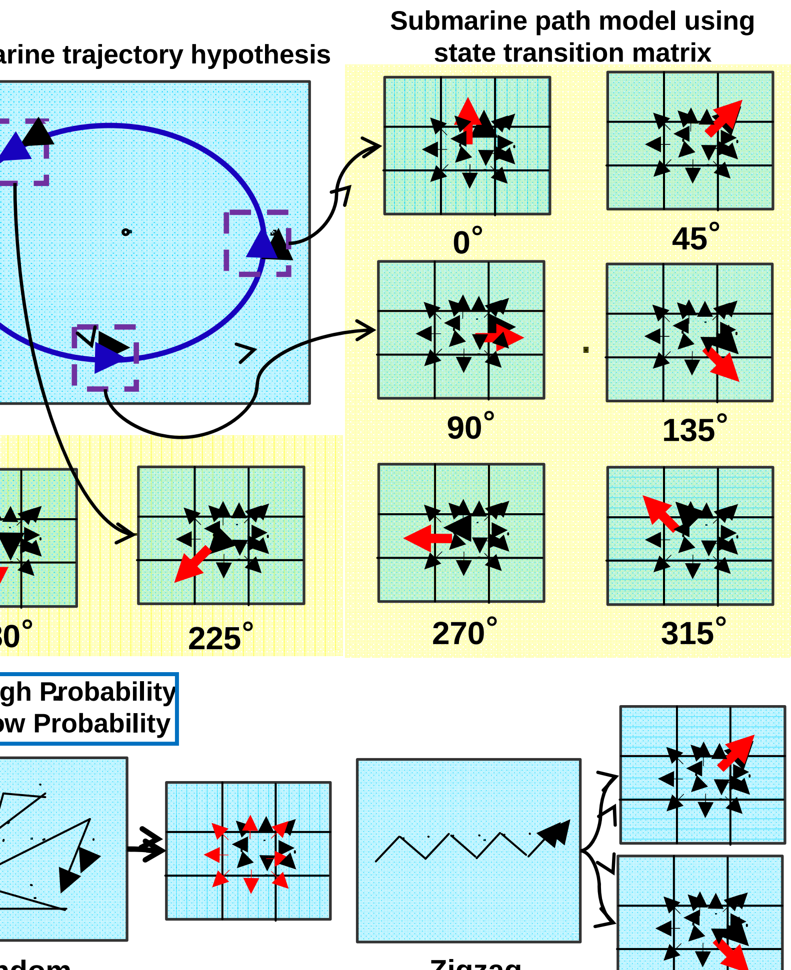
$$B = \left[b_{glij}\right] = \left[P(y(k) = l \mid (x(k) = i, u_g(k) = j))\right]$$

HMM-based Asset Allocation / Search Planning - 2

bmarine Trajectory → Transition Matrix

ansition matrix $A = \left[a_{ij} \right] = \left[P(x(k+1) = j \,|\, x(k) = i)) \right]$

For each cell, choose the cell transition model relevant submarine trajectory hypothesis



■ Active Sonar Equation

$$\boxed{SE = (SL - TL_{ST} - TL_{TR} + TS) - (NL - DI) - DT}$$

SL: Source level, DI: Directivity index
SE: Signal excess, DT: Detection threshold
TS: Target strength, NL: Noise level
TL_{ST}: Transmission loss from source to target
TL_{TR}: Transmission loss from target to receiver

■ Emission Probability Model

- Given the environmental parameters (wind, shipping density, wave heights, etc), tailored products (e.g., Wentz Curve, etc) are used to determine the noise level (NL)

$P(y(k) = 1 \,|\, x(k) = 1) = P_D$ → Detection

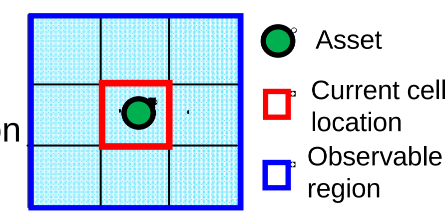
$P(y(k) = 0 \,|\, x(k) = 1) = 1 - P_D$ → Missing

$P(y(k) = 0 \,|\, x(k) = 0) = 1 - P_F$ → No target

$P(y(k) = 1 \,|\, x(k) = 0) = P_F$ → False alarm

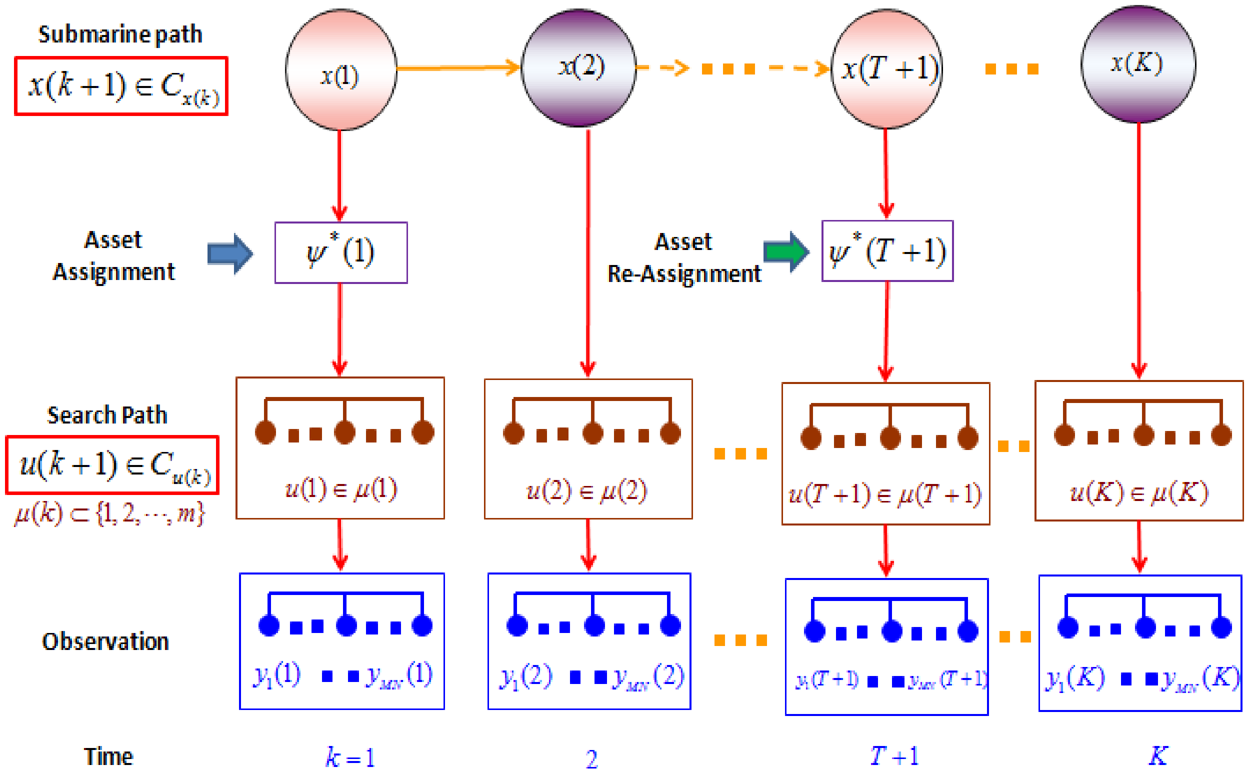
If non-observable $\Rightarrow y(k) = \varnothing$ → Null observation

$$y(k) = \begin{cases} 1: \text{Detection} \\ 0: \text{Non-Detection} \end{cases}$$



$$x(k) = \begin{cases} 1: \text{if submarine = cell } j \text{ = asset} \\ 0: \text{Otherwise} \end{cases}$$

HMM-based Search Problem



- *n* sensors are assigned to *N* X *M* cells to search for a moving target (partially observable restless bandit) → modeled using a HMM
- ntegrated asset allocation and path planning
  - Given total search time, define the search region and the search path for each asset
  - Assignment ⇒ Search path ⇒ State-dependent periodic re-assignment
- Objective function – maximize detection probability of target

Mathematical Problem Formulation

$$\max_{\psi} \sum_{v=0}^{(K/T)-1} \sum_{t=1}^{T} \sum_{i=1}^{m} \sum_{j \in A_i(Tv+1)} \sum_{h \in D_j^i} \pi_h(k \mid k-1) b_{jlri}^h(k) \psi_{ij}(k)$$

where $k = Tv + t, v = 0, \ldots, (K/T)-1,\ t = 1, \ldots, T$

$$\text{s.t.} \sum_{i=1}^{m} A_i(Tv+1) = A, A_i \cap A_j = \varnothing, i \neq j, v = 0, \ldots, (K/T)-1 \qquad \text{(a)}$$

$$\sum_{j \in A_i(Tv+1)} \psi_{ij}(vT+1) = 1,\ \forall i,\ v = 0, \ldots, (K/T)-1, \qquad \text{(b)}$$

$$\sum_{j \in A_i(Tv+1)} \psi_{ij}(vT+t) - \sum_{C_j \in A_i(Tv+1)} \psi_{iC_j}(vT+t+1) = 0,\ \forall i,\ t = 2, \ldots, T-1,\ v = 0, \ldots, (K/T)-1, \qquad \text{(c)}$$

$$\sum_{C_j \in A_i(Tv+1)} \psi_{ij}(vT) = 1,\ \forall i,\ v = 1, \ldots, (K/T)-1 \qquad \text{(d)}$$

Asset i should travel to all regions of A_i without crossing over $A_n, i \neq n$ (e)

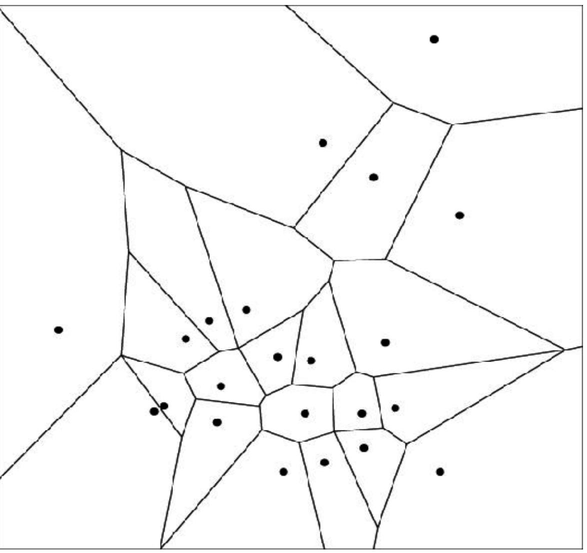
$\psi_{ij}(k) \in \{0,1\},\ i = 1, \ldots, m,\ j = 1, \ldots, MN, k = 1, \ldots, K$ (f)

Constraint (a) → ensures that the assets are assigned to *mutually exclusive search areas* (i.e., no two assets are assigned to the same region)

Constraint (e) → ensures contiguity of search activities.

Constraints (b), (c) and (d) → ensure that the searcher's move is constrained only to the neighboring cells from the current cell within the assigned region

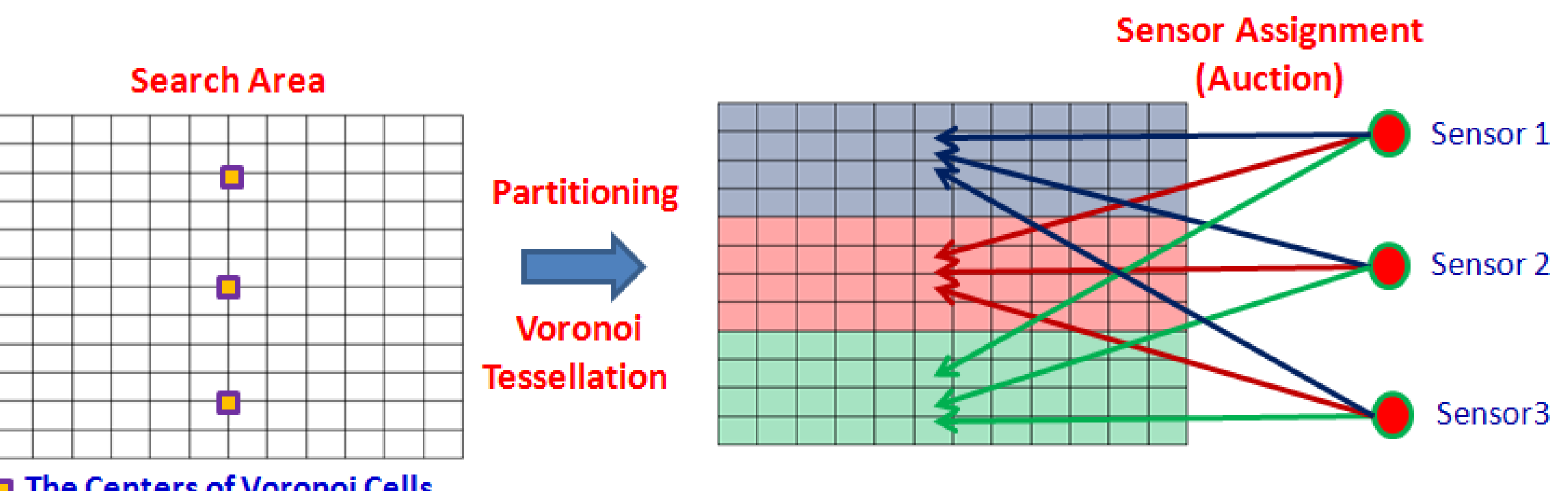
Solution Approach / Phase 1



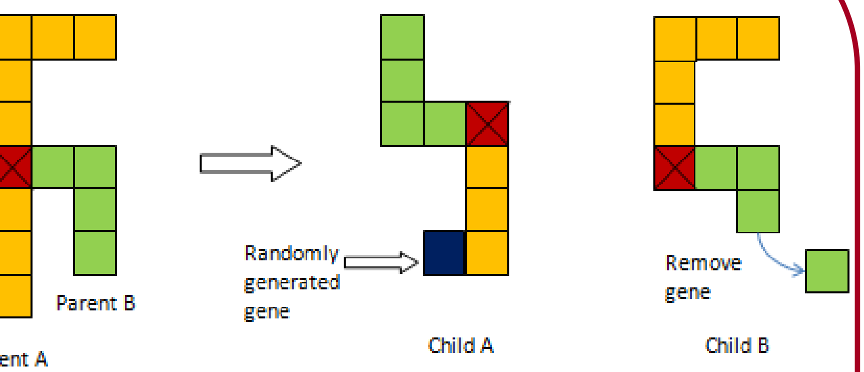
Voronoi tessellation

■ Phase 1: Asset allocation

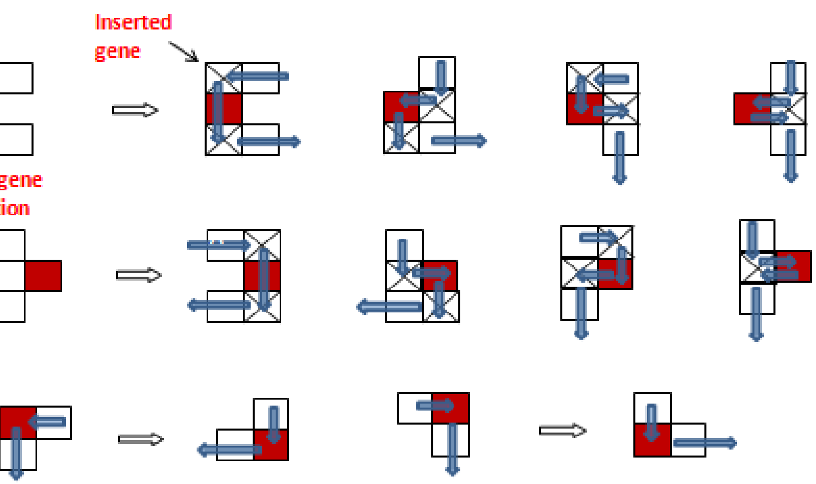
- Employs an evolutionary algorithm, coupled with the Voronoi tessellation approach, for search region partitioning
  - Each individual in the population → sequence of centers of Voronoi cells
- Voronoi tessellation is used for ensuring contiguity and to obtain a near-optimal solution
  - Partition the search space into cells based on some metric (e.g., Euclidean distance)
- Solve the assignment problem of allocating assets to Voronoi cells via the auction algorithm



Solution Approach / Phase 2



ossover process for next generation of search path



Mutation process for next generation of search path

■ Phase 2: Search path

- Solve the optimal search path selection problem via an evolutionary algorithm
  - Each individual in the population → Search path sequence.
- Crossover
  - If parents *A* and *B* are selected for crossover and if they share any common genes, the child can be produced by exchanging the segments of parent A with that of parent *B*
- Mutation
  - A gene is randomly selected and mutated by displacing it from the original location (e.g., left, right, up or down)
  - Additional genes are introduced to maintain a contiguous search path
  - Some genes are removed to satisfy the total search effort constraint

Single Searcher, Single Target Problem - 1

One searcher and one target problem

- Target and searcher move among the 9 cells
- Searcher's move is constrained to 5 move options (i.e., no move, up, down, left, right) or 4 neighboring cells (i.e., up, down, left, right)
- The total search time allotted to each asset is *K*=10 time units
- Submarine motion pattern is modeled via a transition matrix
- The target starts in cell 9
- Detection is certain if the target is in the cell currently being searched

$$P(y(k) = \text{detection} \mid \text{submarine location} = \text{ cell } j = \text{asset location}) = q_j = 1$$

$$V(\pi(k), i) = \text{maximum obtainable probability of detection with } K-k \text{ time periods in cell } i$$

$$V(\pi(k-1), i) = \min_{j \in C_i}\{\underbrace{q_j \pi_j(k)}_{\text{Detection}} + \underbrace{(1-q_j\pi_j(k))V(\pi(k), j)}_{\text{Non-Detection}}\} \Longleftarrow$$

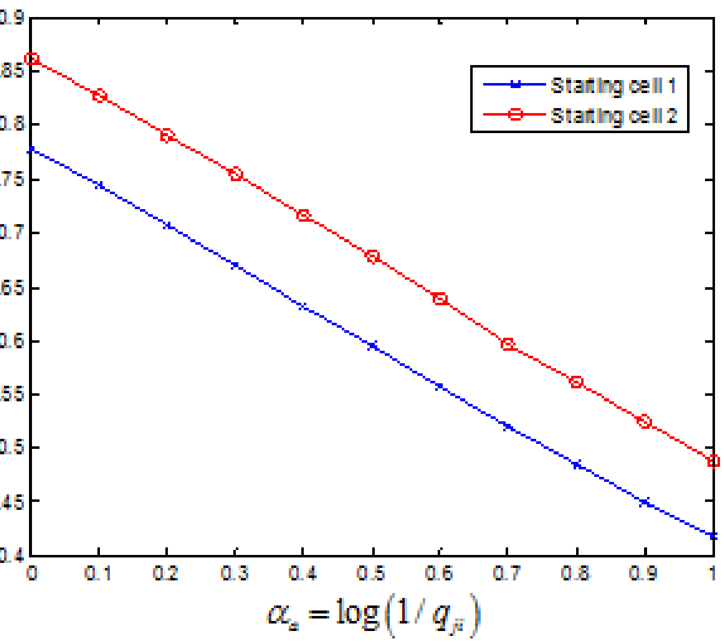
Dynamic programming recursion

1	2	3
4	5	6
7	8	9

$$\begin{bmatrix}
0.4 & 0.3 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0 \\
0.2 & 0.4 & 0.2 & 0 & 0.2 & 0 & 0 & 0 & 0 \\
0 & 0.3 & 0.4 & 0 & 0 & 0.3 & 0 & 0 & 0 \\
0.2 & 0 & 0 & 0.4 & 0.2 & 0 & 0.2 & 0 & 0 \\
0 & 0.15 & 0 & 0.15 & 0.4 & 0.15 & 0 & 0.15 & 0 \\
0 & 0 & 0.2 & 0 & 0.2 & 0.4 & 0 & 0 & 0.2 \\
0 & 0 & 0 & 0.3 & 0 & 0 & 0.4 & 0.3 & 0 \\
0 & 0 & 0 & 0 & 0.2 & 0 & 0.2 & 0.4 & 0.2 \\
0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0.3 & 0.4
\end{bmatrix}$$

Single Searcher, Single Target Problem - 2

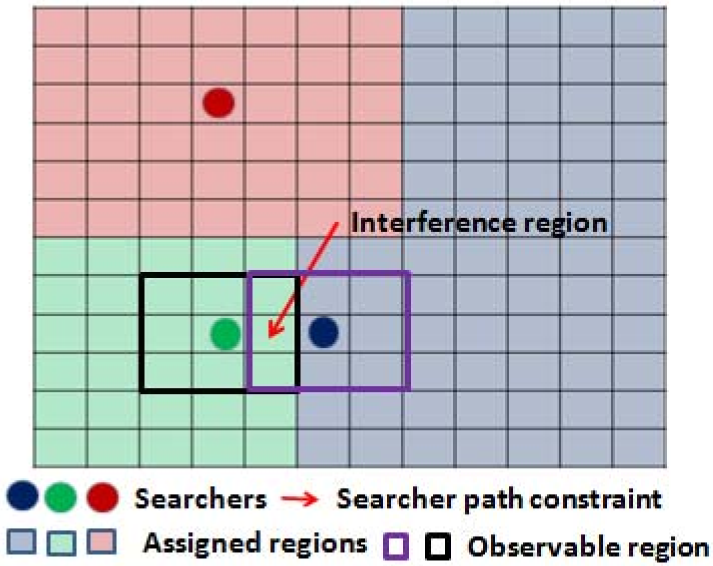
Starting Cell	Optimal Search paths (5 next move options)	Detection Probability	Optimal Search paths (4 next move options)	Detection Probability
1	2 5 6 9 8 5 6 5 8 5	0.7786	2 5 6 9 8 5 6 5 8 5	0.7786
	4 7 8 9 6 5 8 5 6 5	0.7786	4 7 8 9 6 5 8 5 6 5	0.7786
2	3 6 9 8 8 5 6 5 8 5	0.8631	3 6 9 8 5 6 5 8 5 4	0.8615
	5 6 9 8 8 5 6 5 8 5	0.8631	5 6 9 8 5 6 5 8 5 4	0.8615
3	6 6 9 8 8 5 6 5 8 5	0.8631	6 9 8 5 6 5 8 5 6 5	0.8476
4	7 8 9 6 6 5 8 5 6 5	0.8631	7 8 9 6 5 8 5 6 5 2	0.8615
5 , 7	8 8 9 6 6 5 8 5 6 5	0.8631	8 9 8 5 6 5 8 5 6 5	0.8476
6, 8, 9	9	1	9	1



Results

- When the searcher initiates search in cell 1, the optimal detection probabilities are the same in both the cases (4 or 5 move options)
- If the search is initiated in other cells, the detection probability differs in both the cases
- As the sensor capabilities deteriorate (e.g., due to weather), the target detection probability decreases

Three Searchers, Single Target Problem



Interference Model

- Suppose assets are conducting the search activity in non-overlapping partition regions
- **Assumption**: the assets do not observe any valuable information within the interference region
- Observation probability within the interference region is given as:

D^i :Set of observable cells for asset i

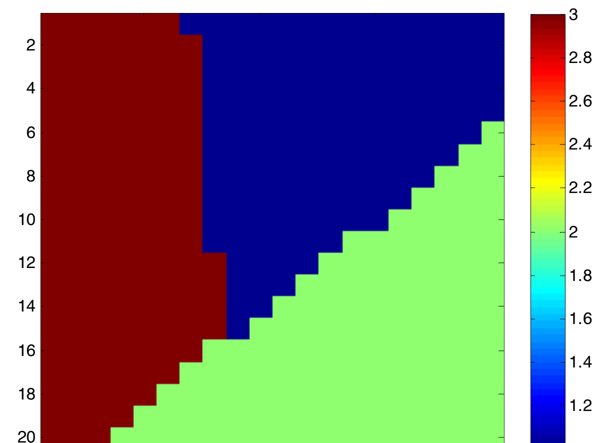
$$\left(D^i \cap D^j \neq \varnothing\right) \Rightarrow \text{Interference region}$$

$$\Rightarrow \text{Non-observable } \left(y(k) = \varnothing\right)$$

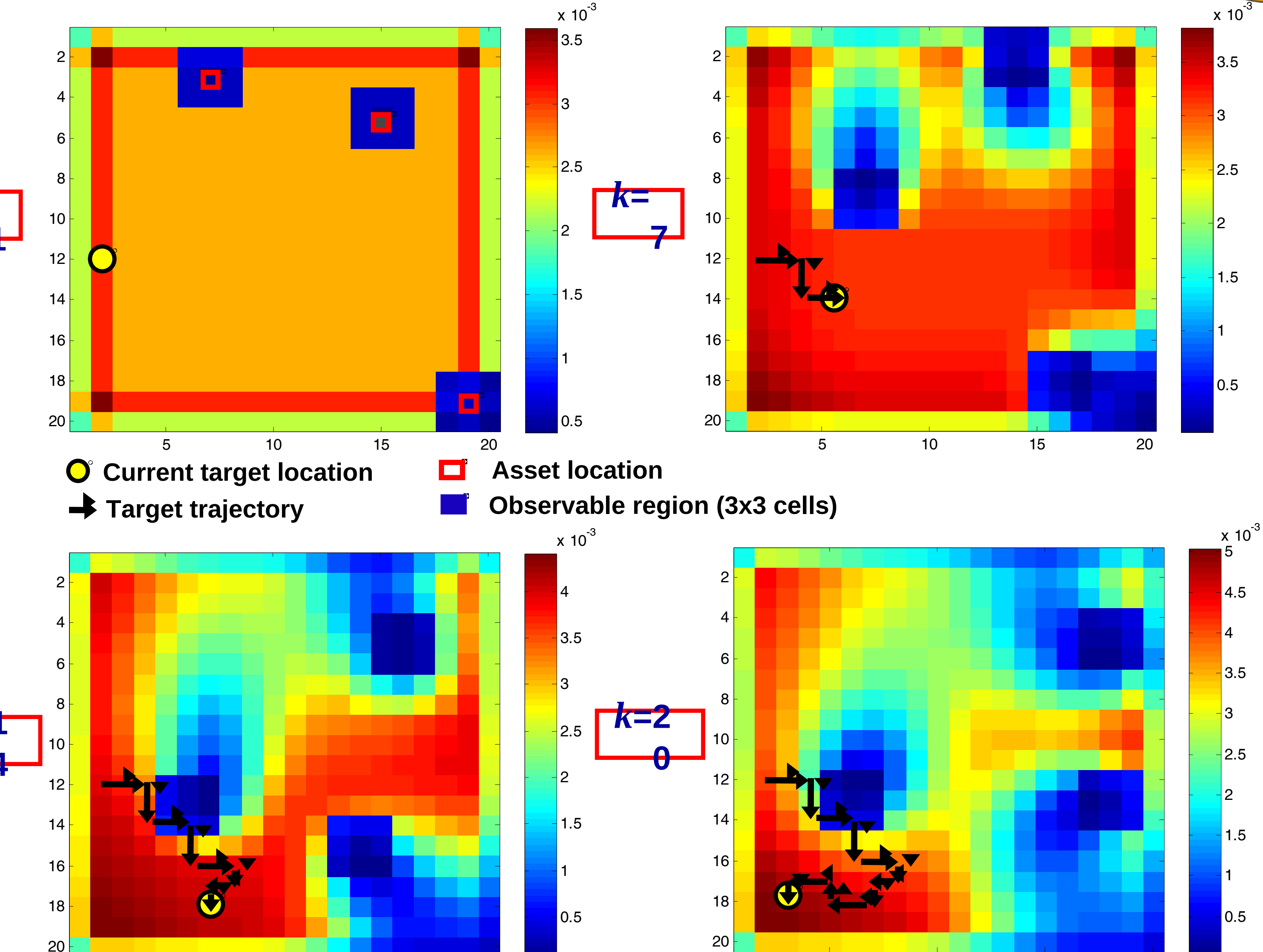
Asset Allocation / Search Planning Example

- Objective: Given an area of 20x20 search cells, allocate three assets of equal search capability to maximize the detection probability
  - Each asset is allotted K=20 time units
  - Searcher's move is constrained only to the neighboring cells (i.e., $\uparrow$, $\downarrow$, $\leftarrow$, $\rightarrow$)
  - Nominal value for SE is used for emission probability setup
  - Animation shows the asset allocation and information state for time units k=1 to 20

Illustrative Search Region Partition



Probability Map of Target Location



Summary

Problem: Moving target search problem using a HMM framework

- Formulation and model of search path problem using a hidden Markov Model

Modeling Approach: Hidden Markov Models

- Asset/sensor capability modeled using emission matrix
- Submarine trajectory modeled using transition matrix

2-Phase Solution Approach

- Phase 1 : Search region partition and asset allocation
- Phase 2 : Search path optimization

Application

- One searcher - one target
- Multiple searchers - one target

Future Work

- Multiple searchers - multiple targets
- Intelligent targets