On Optimizing Command and Control Structures

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Overview

- Information Theory, Graph Theory and C2
- Information Flow Simulation
- Information Theory and Control
- Information Flow with Actuation Simulation
- Summary

Information Theory and C2

- **C2 process** $x(t)$
- **Discrete state space** $X = \{x_i\}$
- **Descriptive complexity (Hartley Information) –** $\log_2 |X|$
- **Uncertainty (Shannon entropy) –** $H(x(t)) = -\sum P(x_i, t) \log_2 P(x_i, t)$
- **Information –** $I(x_i, t) = -\log_2 P(x_i, t)$
- **Information Volume – sum of all information of all entities**
- **Observation –** $S = (\xi, t)$

Information Theory and C2

Information Decay due to Entropic Drag

- **Typically,** $H(x(t')|S) > H(x(t)|S)$ **for** $t' > t$
  - Indicates a loss of information of the observation $S = (\xi, t)$
  - We call this decay *entropic drag*
- **Sequence of observations** $S_{1:k} = \{(\xi_j, t_j)\}$
- **Define the entropic drag**

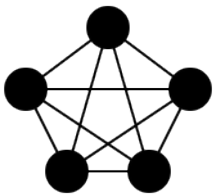
$$\Gamma(S_{1:k}, t_k, t') = \frac{H(x(t')|S_{1:k})}{t' - t_k}$$

Graph Theory and C2

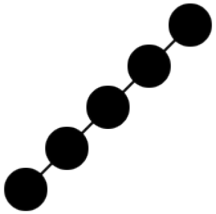
- **Model the various entities and interconnections as a graph $G = (V, E)$**
- **Vertex set V represents entities**
  - Sensors a subset of V and observe $x(t)$
  - Entities share info with neighbors and process info
- **Edge set E represents comm links**
- **Sensors are exogenous comm channels**

APL

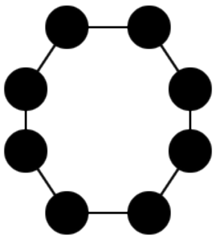
Sample Graph Topologies



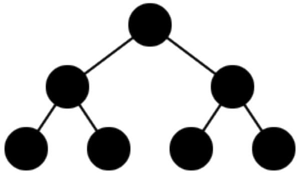
Fully Connected



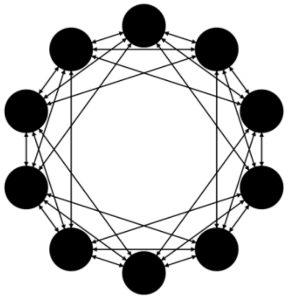
Path



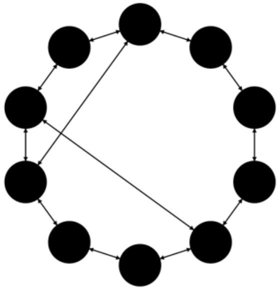
Ring



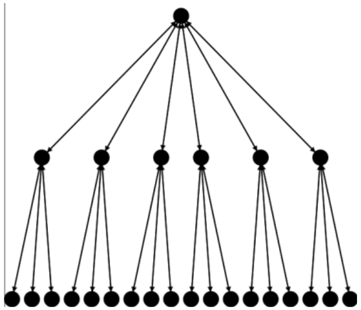
Binary Tree



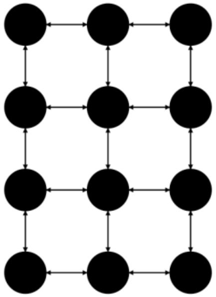
3-Ring



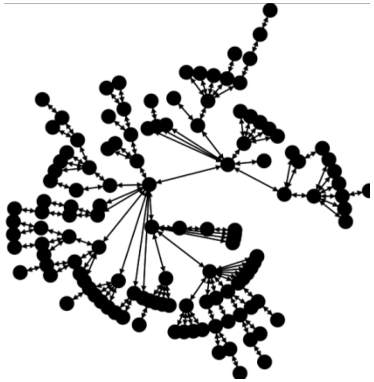
1-Ring+2 (small world)



[6,3] Regular Tree



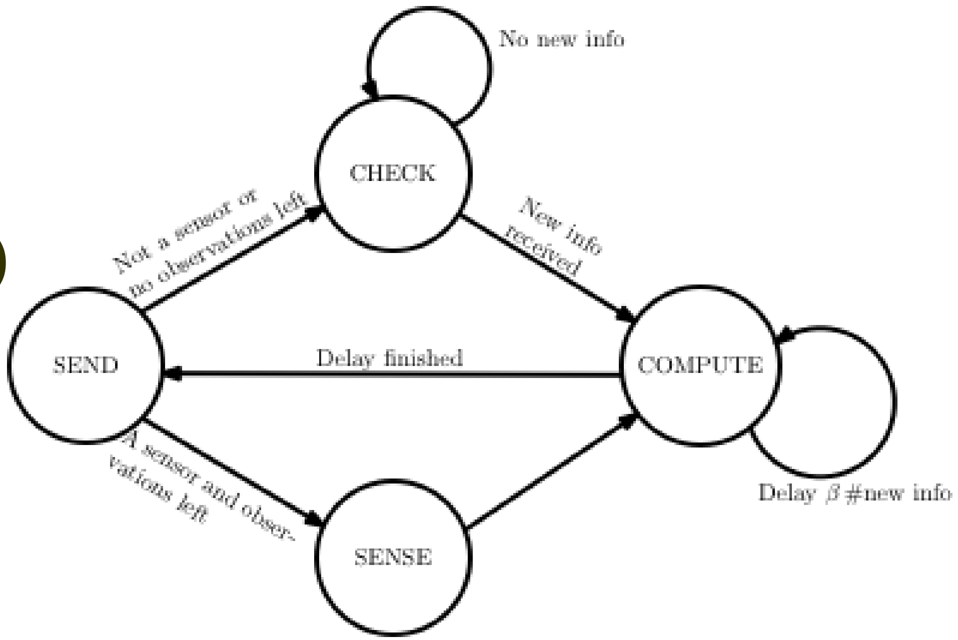
3x4 Grid



(1,1) Scale-free

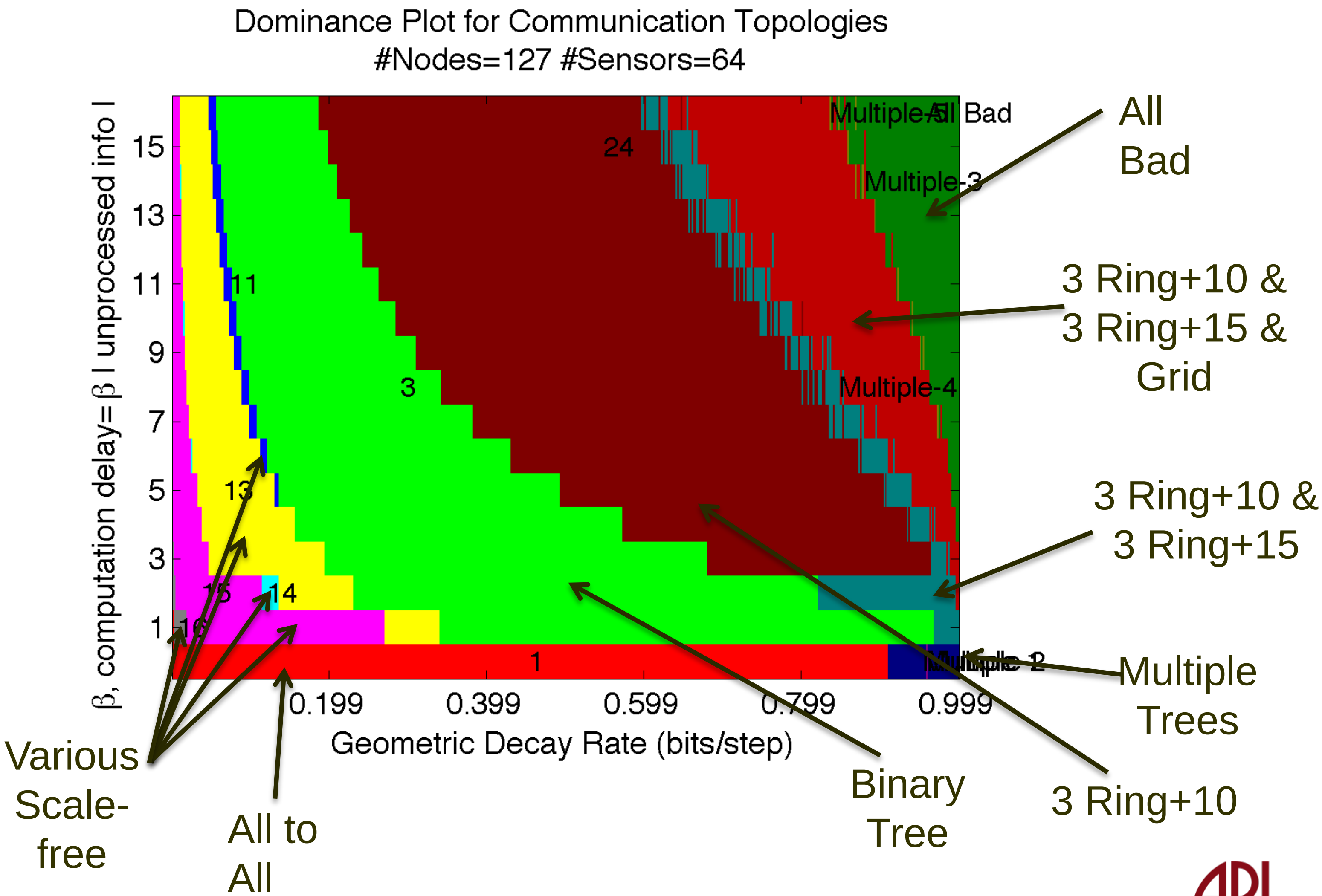
Information Flow Simulation

- **SEND, SENSE, COMPUTE and CHECK states**
- **Computational time (complexity) linearly proportional to new information received (β)**
- **Information decays via a geometric decay rate (Γ)**
- **Investigating peak processed information volume to single observation**



Topology Impact on Situational Awareness

30 topologies compared



Information Theory and Control

- **Control actuations change the state probabilities of the system**
- **(Hopefully) these changes create an increase in order**
  - Called *negentropy*
- **An actuation** A **has negentropy** $I_A(t) = H(x(t)|A)$
- **A sequence** $A_{1:k}$ **has negentropic drag**

$$\Gamma_A(A_{1:k}, t_k, t') = \frac{H(x(t')|A_{1:k})}{t' - t_k}$$

- **Negentropy measures increase in order but not the quality of decision!**
  - This requires the notion of *utility*

Information Flow with Actuation Simulation

- **Extended simulation by adding control**
- **Sensors now sense and actuate**
- **Entities pass information up the hierarchy**
- **Entities pass actuations down the hierarchy**
- **Entities compute intermediate actuations for the actuators below them in the hierarchy**

Information Flow with Actuation Simulation

- **Only trees compared, but multiple sensor waiting behaviors**
  - Idle, re-sense, and re-sense+re-process
- **All graphs had 64 sensors, but varying intermediate nodes**
- **Using total information behind actuations as metric**
- **Results indicate different regions of decay rate where different topologies and sensor behaviors outperform others**

Summary

- **Information theory can be used to characterize C2 processes**
- **Graph theory is a natural model for C2 interconnections**
- **Entropic drag and computational complexity help determine optimal C2 topologies**

Related Work at JHU/APL

- **Extensions – fusion/truncation, processing models, gossip, abstraction, etc.**
- **Genetic algorithm to find better topologies**
- **Agile controllers**
- **Information-theoretics of human/AUV coalitions**
- **Counter-C2**