On the ‘Boyd- Kuramoto Model’: Emergence in a Mathematical Model for Adversarial C2 Systems

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C2 Processes: many are cycles!

- **Boyd’s Observe-Orient-Decide-Act Loop:**

- **Snowden’s Cynefin Framework:**
  Different loops depending on context

- **Operational Planning Process**

- **Elaborations:**
  Lawson’s C2 Cycle; DOODA, …

Interacting OODA:
Moon, Kruzins, Calbert 2002
The Kuramoto* Model

\[ \dot{\theta}_i = \omega_i + \frac{K}{N} \sum_j A_{ij} \sin(\theta_j - \theta_i) \]

Kuramoto Model - 1-dim Oscillator:

- Natural Frequency
- Coupling
- Network adjacency matrix

Phase Synchronisation at High K

Incoherence at Low K

† Kalloniatis, Phys. Rev. E 82, 066202, 2010
Also contributions by R. Taylor and T. Dekker
Mapping Kuramoto to Boyd

\[ \theta = \text{Point of progress in decision cycle.} \]

\[ K = \text{Coupling} = \text{degree of tightness of control.} \]

\[ \omega = \text{Natural frequency of each node} = \text{inverse time period for processing appropriate information according to "environment" in order to advance through cycle.} \]

\[ A = \text{intra- C2 Network} = \text{not just communications connectivity, but also authority, collaborative, social, and visual networks.} \]

- \text{Who are my points of reference for my decision cycle?}
- \text{With whom must I mutually adjust to progress decisions?}

Periodicity of \text{sine} response function: irrelevance of “stale” information or past decisions: the \text{current decision cycle is all that matters.}
Modern military operations involve diverse time scales and networking of processes.

Model in this paper: only one such echelon included.
The ‘Boyd-Kuramoto’ equations
cf Lanchester attrition and Hughes salvo equations

Phase angles
Intrinsic frequencies
Adjacency matrices

N.B. This is a Caricature:
Informal Networks in Traditional Military (Ali 2011); Hierarchy in Insurgent Networks (Memon et al 2008)

\[
\begin{align*}
\dot{\beta}_i &= \omega_i + \sigma_B \sum_{j=1}^{N_B} B_{ij} \sin(\beta_j - \beta_i) + \zeta_{BR} \sum_{j=1}^{N_{BR}} M_{ij} F(\rho_j - \beta_i) \\
\dot{\rho}_i &= \nu_i + \sigma_R \sum_{j=1}^{N_B} R_{ij} \sin(\rho_j - \rho_i) + \zeta_{RB} \sum_{j=1}^{N_{BR}} M_{ij} G(\beta_j - \rho_i).
\end{align*}
\]

\(\omega_i, \nu_i \in [0,1]\) uniform random distribution interactions only within one ‘echelon’
Intelligence- Surveillance- Reconnaissance & OODA

Blue has total ISR

Blue seeks to ‘get inside adversary OODA loop’

\[ F(\rho_j(t) - \beta_i(t)) = \sin(\rho_j(t) + \lambda - \beta_i(t)) \times 1 \]

\[ \lambda = \pi/4 \approx 0.7854 \]

Red synchronises around

Blue `Actions’ with narrow ISR

\[ G(\beta_j(t) - \rho_i(t)) = \sin(\beta_j(t) - \rho_i(t)) \exp\left(-\left(\beta_j(t)\right)^2 / 2s^2\right) \]

\[ s = \sqrt{\pi} \approx 1.772 \]
Measures of Performance

Measure of internal synchronisation
(B↔B, R↔R)

\[ r(t)e^{i\Psi(t)} = \frac{1}{N} \sum_{i} e^{i\theta_i(t)} \]

\[ r_B(t) = \frac{1}{N_B} \left| \sum_{i} e^{i\beta_i(t)} \right| \]

\[ r_R(t) = \frac{1}{N_R} \left| \sum_{i} e^{i\rho_i(t)} \right| \]

Measure of external synchronisation
B↔R

\[ \Delta_{BR}(t) = \frac{1}{N_{BR}} \sum_{i} \left[ \beta_i(t) - \rho_i(t) \right] \]
Basic (Extreme) Behaviours

Blue focused exclusively on Red; neither internally coordinates.

\[ \sigma_B = 0 \quad \zeta_{BR} = 30 \]
\[ \sigma_R = 0 \quad \zeta_{RB} = 0 \]

Blue, Red focused exclusively on internal coordination but no regard for each other

\[ \sigma_B = 0.8 \quad \zeta_{BR} = 0 \]
\[ \sigma_R = 0.15 \quad \zeta_{RB} = 0 \]

For this instance!

\[ \bar{\omega} = 0.56 \]
\[ \bar{\nu} = 0.61 \]
Emergence: the ‘surprise’

Laughlin: “system qualities or behaviours not reducible to the system components but arise from their interactions.”

Eg. Random selection of frequencies makes nodes ‘close’ even though topologically ‘far’; creates affinity for dynamically forming a sub-cluster.

This cannot be designed for given agent differentiation. Sub-clusters ‘emerge’ at intermediate interaction strengths. Each instance is different!
Blue v Red at the Edge of Chaos

\[ r \]

\[ \sigma_B = 0.6 \quad \zeta_{BR} = 0.1 \]
\[ \sigma_R = 0.075 \quad \zeta_{RB} = 0 \]

\[ r \]

\[ \sigma_B = 0.6 \quad \zeta_{BR} = 0.8 \]
\[ \sigma_R = 0.075 \quad \zeta_{RB} = 0 \]

\[ r \]

\[ \sigma_B = 0.6 \quad \zeta_{BR} = 2 \]
\[ \sigma_R = 0.075 \quad \zeta_{RB} = 0 \]

Blue becomes more synchronised through stronger coupling to Red at cost of oscillations in staying inside Red’s OODA

\[ \text{DID YOU EXPECT THAT?} \]
Another example: Only Red ‘at the Edge’

\[ \sigma_B = 0.6 \quad \varsigma_{BR} = 0 \]
\[ \sigma_R = 0.08 \quad \varsigma_{RB} = 0 \]

\[ \sigma_B = 0.6 \quad \varsigma_{BR} = 0.2 \]
\[ \sigma_R = 0.08 \quad \varsigma_{RB} = 0 \]

\[ \sigma_B = 0.6 \quad \varsigma_{BR} = 0.6 \]
\[ \sigma_R = 0.08 \quad \varsigma_{RB} = 0 \]

\[ \sigma_B = 0.6 \quad \varsigma_{BR} = 2 \]
\[ \sigma_R = 0.08 \quad \varsigma_{RB} = 0.1 \]

Blue adapts to red at cost of zig-zag in staying inside Red’s OODA
Conclusions

There are more variables by which traditional C2 structures can achieve Agility; they are subject to mathematical modelling.

Model enables finding the balance point for given C2 structures and time scales between internal coordination and responsiveness to adversary.

Emergence is nothing mystical: mathematical models can capture such surprises in representations of C2.

Applications: realistic network data, human factors data, limited/interrupted ISR functions also for Blue.

Multi-echelon, multi-time spectrum: needs modification of equations.

‘Boyd- Kuramoto’ can intermediate/cross-validate between high/low fidelity models of C2 systems.

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