# An Application of Information Theory to Command and Control

Kevin Schultz and David Scheidt
Applied Physics Laboratory
Johns Hopkins University

# Information Theory

Continuing the development in (Scheidt & Schultz, 2011),

- C2 Process $x(t)$
- Discrete state space $X = \{x_i\}$
- Uncertainty (Shannon entropy) $H(x(t)) = -\sum P(x_i, t) \log_2 P(x_i, t)$
- Observations, $S = \{(\xi, t)\}$, and sequences of observations $S_{1:k} = \{(\xi_j, t_j)\}$
- Entropic drag $\Gamma(S_{1:k}, t_k, t') = \frac{H(x(t')|S_{1:k})}{t' - t_k}$
- Can "close the loop" using analogous constructions for the uncertainty removed by control events (actuations)

# Example

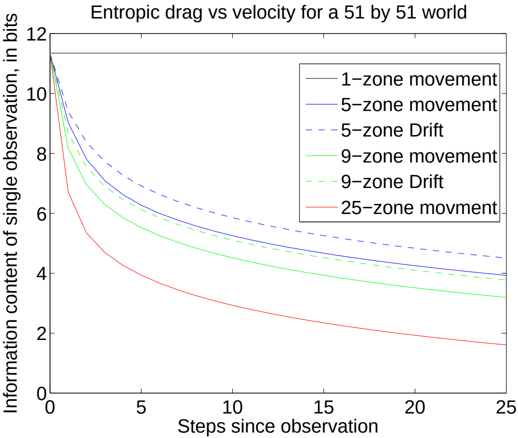
Consider the following scenario to illustrate entropic drag:

- Single target tracking scenario, discrete time and space
- State transitions define motion models
  - $x_j \to x_i$ with known probability $P_{x_j}(x_i)$
- Given $P(x_i, k)$ have $P(x_i, k+1) = \sum_{x_j \in X} P(x_j, k) P_{x_j}(x_i)$
- Can repeatedly reply this relationship to calculate probability distribution at future times
- Several motion models considered
- Calculating the information loss of an initial observation due to entropic drag

APL
JOHNS HOPKINS UNIVERSITY
Applied Physics Laboratory

# Results

- Motion models that introduce more uncertainty at each time step have greater entropic drag and more information loss
- Different models represent different levels of target agility
- Time can elapse due to slow sensor sampling rate, or relatively faster adversarial agility



# Relevance

- Demonstrates that entropic drag is a real phenomenon that can be numerically calculated
- This example is a *hidden Markov model*, which is well studied in state estimation and control
  - Kalman , particle, and PHD filtering; multi-hypothesis tracking, etc.
- Hidden Markov models commonly employed to estimate both state and uncertainty
  - This provides insight on how to estimate the entropic drag from the system from the existing state estimation scheme
  - Use this estimation of entropic drag to configure the state of an *agile* control system

# Abstraction

- Want to consider the effects of summarizing multiple elements of interest into a more abstract entity in a C2 scenario
  - e.g., soldiers into squads, squads into platoons,. . .
- Hypothesis: information-theoretic characteristics of the C2 system determine the optimal level of abstratction

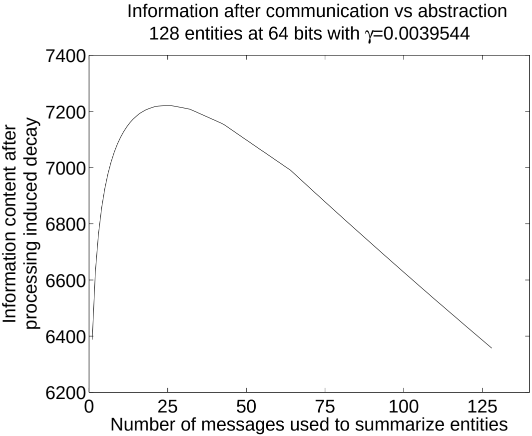
# Model

To model abstraction, consider the following:

- Have $N$ elements of interest
- Described using messages of length $L$ bits with channel rate $r$
- Want to summarize using $M \leq N$ elements
- Approximate information loss due to abstraction by $I(M) = N \cdot L - \lfloor N/M \rfloor \cdot \log_2(M^2/N^2)$
- Also want to consider effects of entropic drag during the communication and computation time
  - Assume geometric decay rate $\gamma \in [0, 1]$
  - $f(M, L, r)$ determines the total communication and computation time
- $\tilde{I}(M, \gamma) = I(M) \cdot (1 - \gamma)^{f(M,L,r)}$

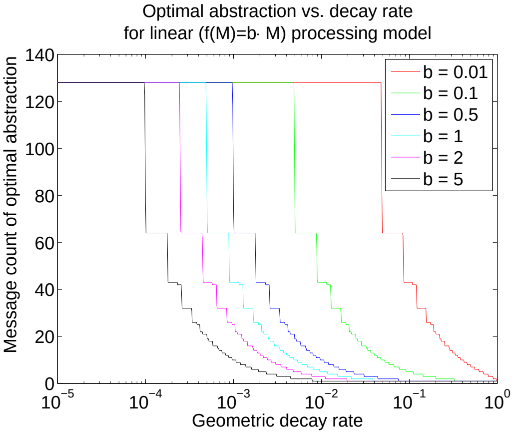
# Results

- Optimal level of abstraction (in terms of total information after communication) depends on information decay rate $\gamma$
- Complexity ($f(M, L, r)$) also affects the optimal level of abstraction



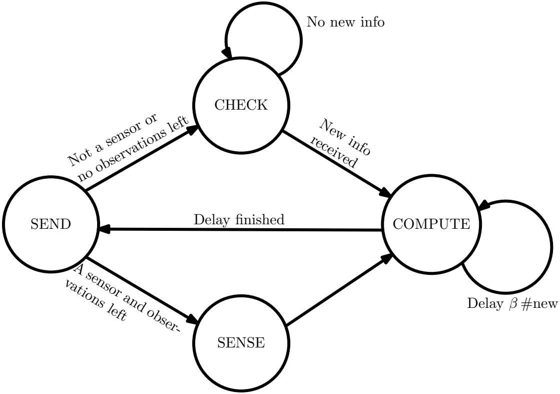
# Results

- Optimal level of abstraction (in terms of total information after communication) depends on information decay rate $\gamma$
- Complexity $(f(M, L, r))$ also affects the optimal level of abstraction

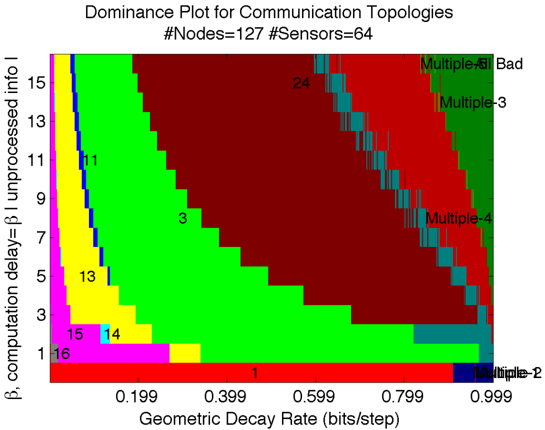


# Simulation Overview

- SEND, SENSE, COMPUTE, and CHECK states
- Computational time (complexity) linearly proportional to new information received ($\beta$)
- Information decays via a geometric decay rate ($\Gamma$)
- Investigating peak processed information volume to a single observation

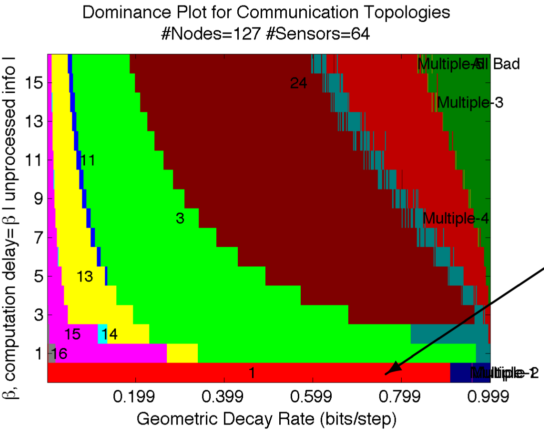


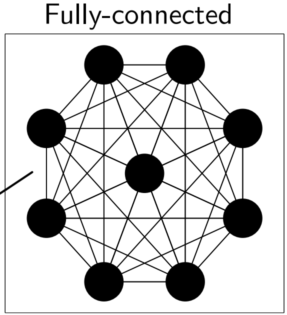
# Initial Results



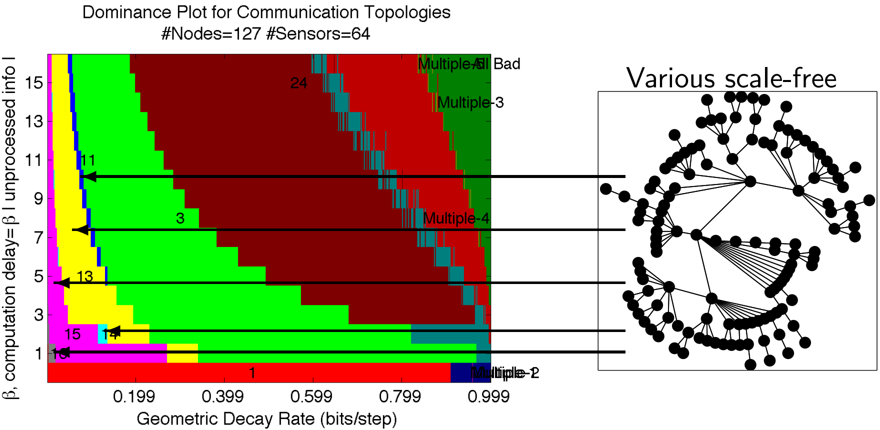

Dominance plot showing which topology accumulated the most information as the information-theoretic characteristics changed.

# Initial Results

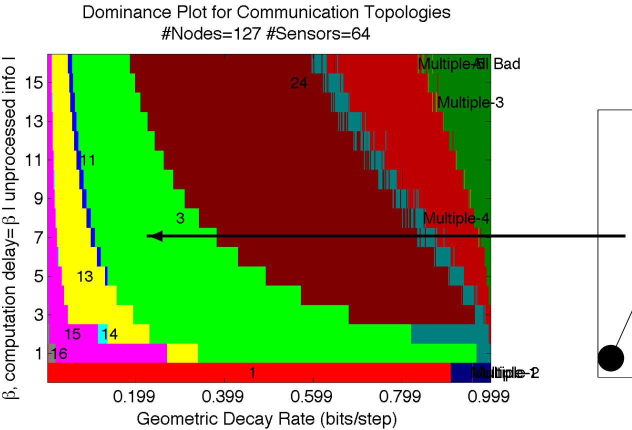


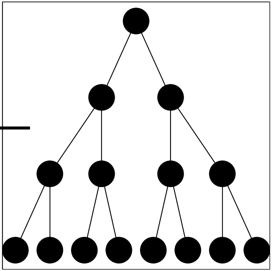
# Initial Results



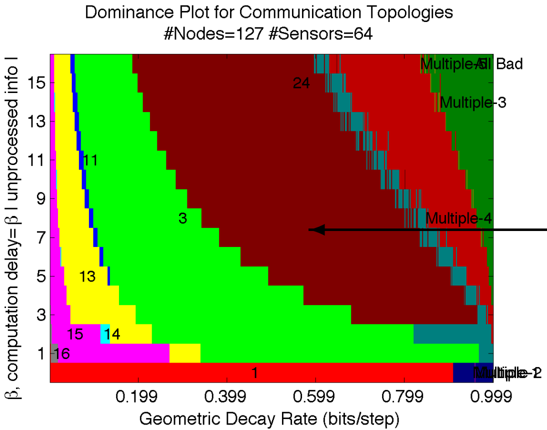
# Initial Results

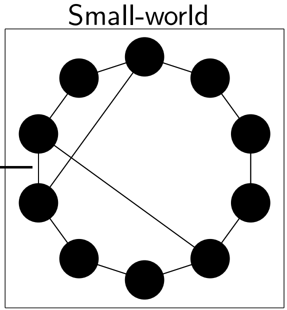



Binary tree

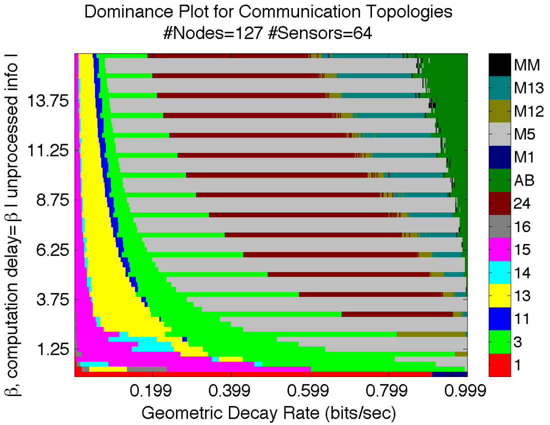


# Initial Results



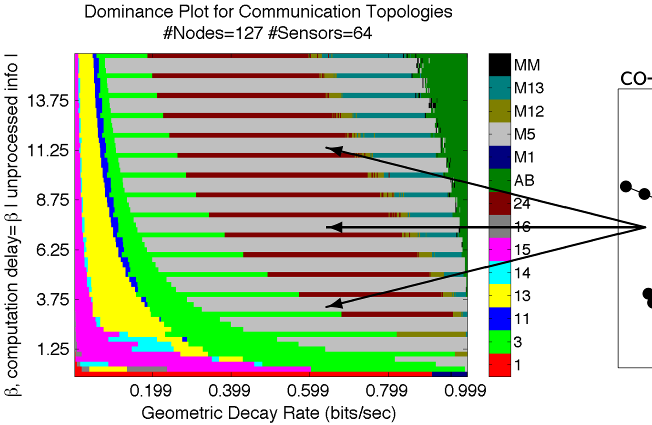



# Additional Complexity

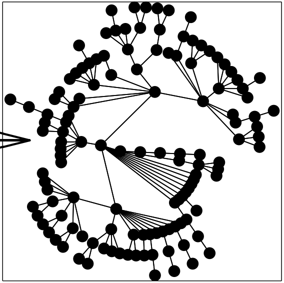



When non integer complexity values considered, additional structure is added to the dominance plot

# Additional Complexity



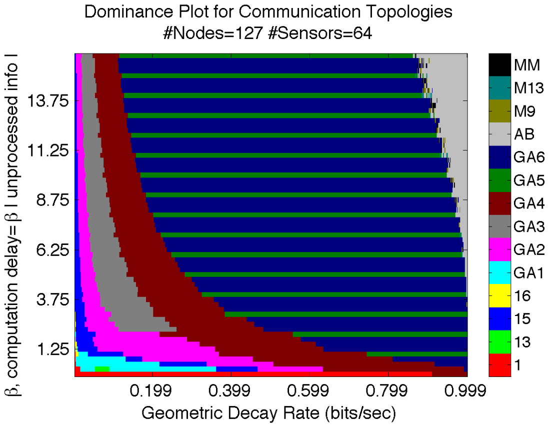

Scale-free trees co-dominant in bands



# Genetic Algorithm Optimization

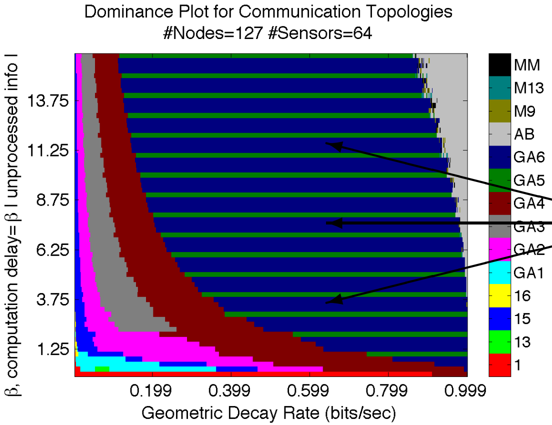
- In (Scheidt & Schultz, 2011) 30 different topologies were compared over the information-theoretic parameter space
  - Certain topologies were dominant in different regions
  - Certain categories of topologies were dominant near each other
- Here, investigate the possibility of optimizing topology directly at a given point in the parameter space
- Using a genetic algorithm, the topology of the graph was optimized at seven specific points
- Also considered multi-point optimizations

# Results

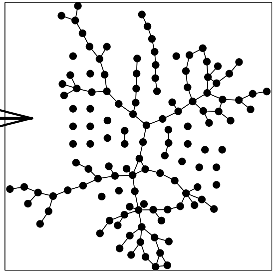



The seven optimized topologies perform well in nearly the same regions as the topologies they improved upon

# Results




Example: GA6 optimized in the banded region



# Results

- It is possible to optimize for specific information theoretic parameters
  - This optimization is robust, in that it performs well in a neighborhood
- It is also possible to find a few optimized topologies that outperform original set of 30
  - So a look-up table approach may be feasible when direct optimization is not
- Analysis of multi-point optimizations provides insight into behavior of simulation

JOHNS HOPKINS UNIVERSITY
Applied Physics Laboratory

# Future Work

- Information-theoretic characterization of real systems
  - UxV control
  - Ship auxiliary systems
- Information-theoretic characterization of human performance
  - Understanding mixed human/machine performance
- Design of agile systems

# Questions?

This work was supported by Office of Naval Research contract N00014-0-C-0396.