An Analytical Model That Provides Insights into Various C2 Issues

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Overview

- Model of Parallel Acquisition of Targets
  - Kill Rate Consequences
    - Taylor’s New General Methodology for Lanchester Attrition-Rate Coefficients
    - Analytical Expression for Kill Rate
      - Higher Kill Rate Than for Serial Acquisition
  - Insights into Network-Centric Warfare
Effects of Parallel Acquisition

- More Efficient Target Acquisition
- Force Multiplier
  - Inflate More Casualties on Enemy
  - Sustain Fewer Casualties
- Example
  - X Force Can Effect Change from Serial to Parallel Acquisition of Targets
Basic Lanchester-Type Paradigm

\[ \frac{dx}{dt} = -a \ y \quad \text{with} \quad x(0) = x_0 \]

\[ \frac{dy}{dt} = -b \ x \quad \text{with} \quad y(0) = y_0 \]

Simplified Representation
Lanchester Attrition-Rate Coefficients

- $a$ and $b$ are called **Lanchester Attrition-Rate Coefficients**

- $a =$ Rate at Which an Individual Y Firer Kills X Targets (Single-Weapon-System-Type Kill Rate); Kill Rate of Single Typical Firer
Kill-Rate Model

- Considers Single Typical Firer against Passive Target
  - No Consideration of Duel
- Does Not Consider Effect on Target by Any Other Firer
  - Can Develop Correction Factor to Account for Such Effects
New General Methodology for Lanchester Attrition-Rate Coefficients

- Recently Developed by Taylor
- Greatly Expands Modeling Capabilities
  - Great Detail in Target-Engagement Cycle
    - Special Treatment of First Round(s)
    - Actual Distributions for Event (e.g. Interfiring) Times
  - Battle Damage Assessment
  - Command & Control at Platform Level
  - Insights into Network-Centric Warfare
Conditions Considered

- Heterogeneous-Target Environment
- Stochastic Line of Sight (LOS)
- Target-Acquisition Times Independent (But Otherwise Arbitrary)
- Interfiring Times Independent (But Otherwise Arbitrary)
New Methodology

- **Kill Rate Computed as Ratio of**
  - *Expected Number of Kills in Target-Engagement Cycle to*
  - *Expected Duration of Target-Engagement Cycle*

\[
a_{ij} = \frac{n_{cyclek_{X_iY_j}}}{t_{cycleY_j}}
\]
Can Now Model

- In Tank Warfare
  - First Round Chambered
  - Tank Commander Acquires Targets While Gunner Engages
  - Automatic Loader (in Russian Tanks)

- Information Aspects
  - Battle Damage Assessment
    - Time to Assess
  - False Targets
Conditions in Specific Cases

- Heterogeneous-Target Environment
- Stochastic Line of Sight (LOS)
- Target-Acquisition Times Exponential (and Independent)
- Interfiring Times Exponential (and Independent)

Can Be Extended to Log Normal/Erlang Times
Key Question

- Can New Targets Be Acquired While an Acquired Target Is Being Engaged?
- Simplest Model Considers Two Cases
  - No New Target Can Be Acquired
    - Serial Acquisition
  - New Target Can Be Acquired (At Same Rate)
    - Parallel Acquisition
Target-Engagement Cycle
(Parallel Acquisition of Targets)
Some Computations

- Y Always Uses Serial Acquisition
- X Can Change from Serial Acquisition of Enemy Targets to Parallel Acquisition
  - Computations Done for These Two Cases
    - Serial Acquisition by X
    - Parallel Acquisition by X
Both Sides Serial
(Force-Level Decays)

\begin{align*}
\text{Force Level} & \quad \text{Time, } t \text{ (hours)} \\
\text{X Force Level} & \\
\text{Y Force Level} & \\
\end{align*}

(Rates in hours)
\begin{align*}
\text{Single-Target} & \\
\text{Acquisition Rate} & 0.1 \quad 0.07 \\
\text{Conditional Kill Rate} & 8.0 \quad 1.5 \\
\text{Probability LOS} & 0.9 \\
\text{Rate of Losing LOS} & 0.01 \\
\end{align*}
Combat Model
(Only X Force Uses Parallel Acquisition)

\[
\frac{dx}{dt} = - \frac{P_{K(LOS)_{XY}} y}{P_{LOS} \lambda_{XY} x + \frac{1}{\phi + \mu}}
\]

\[
\frac{dy}{dt} = - \frac{P_{K(LOS)_{YX}} x}{P_{LOS} \lambda_{YX} y + \frac{1}{\phi + \mu}}
\]

Intermittent LOS
Target Availability

- Typical X Firer Keeps on Continuously Acquiring Targets from Beginning of Battle

- Target Availability Given by (Assuming Steady State for LOS Process and No Targets Initially Acquired)

\[ B_{tg} = P_{LOS} \frac{\lambda_{YX}}{\lambda_{YX} + \mu} \left\{ 1 - e^{-b_{YX} + \mu g} \right\} \]
Effects of Changing from Serial to Parallel Acquisition

Intermittent LOS
Benefits to X

- Inflicts 62% More Attrition
- Suffers 19% Less Attrition
- Turns Defeat into Victory
Final Comments

- Significant Benefits from Parallel Acquisition Demonstrated for Combat at Platform Level
- Ideas Can Be Adapted to Modeling Network-Centric Warfare
- Such Analytical Models Very Convenient for Showing Benefits from Network-Centric Warfare