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THE STATE OF THE ART AND THE STATE OF THE PRACTICE

A STATE-SPACE FORMULATION FOR EFFECTS-BASED OPERATIONS

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ABSTRACT

The paper describes a general state-space representation of the key concepts for Effects-Based Operations (EBO). The primary purpose is to provide a coherent framework for analytical support for the EBO functions of Knowledge Base Development, Effects-Based Planning, Execution and Assessment; as well as representation of the desired operational end-state, operational design and the associated measures of performance and effectiveness. The formulation includes both differential and difference equation representations with reference to control system analogies. One of the main motives behind the state-space formulation is the explicit representation of uncertainty in the dynamic evolution of the effects achieved and the observation and assessment of the operation.

OUTLINE

NATO, through the Experimentation Program of the Allied Command Transformation (ACT), is conducting experiments with doctrine and tools for effects-based operations (EBO). The ACT development of NATO EBO concepts is closely coordinated with the US Joint Forces Command (JFCOM), which leads the overall multinational EBO concepts development effort. The next major event in a series of exercises is the Multinational Experiment 4 (MNE-4) in February and March 2006. The main NATO C3 Agency (NC3A) contribution to this is the EB-TOPFAS (Effects-Based Tool for Operational Planning, Force Activation and Simulation). EB-TOPFAS is the planning tool for effects-based planning (EBP). The other main functions of EBO are effects-based execution (EBE) and effects-based assessment (EBA). All three functions (EBP, EBE and EBA) depend on and contribute to the fourth main function, the knowledge base development (KBD). Doctrine and tools for all four functions will be tested in MNE-4.

Although it may be said that all good commanders have always conducted effects-based operations it is equally clear that the current development and formalization of EBO concepts extends the traditional framework for the planning and conduct of military operations. This is directly represented by extending the operational space to include all dimensions of the political, military, economic, social, infrastructure and information (PMESII) environment. The increase in dimensionality necessarily increases the complexity of the planning and execution with the associated need for extended expertise in all PMESII dimensions and analytical support to tie the strands together into a coherent operational design.

The purpose of this Note is to provide a coherent analytical framework for EBO by mapping the EBO functions and concepts to well-known engineering formulations and techniques from the disciplines of state-space analysis and optimal control system design. The state-space formulation includes both differential and difference equation representations with reference to control system analogies. One of the main motives behind the state-space formulation is the explicit representation of uncertainty in the dynamic evolution of the effects achieved and the observation and assessment of the operation. The formulations are general and are not expected to provide the capability to “calculate” operational designs in the near term. The dimentionality and complexity of EBO prevents this. However, as lessons learned from the application of EBO concepts evolve, the general formulations should be replaced by more specific quantified relationships of demonstrated or postulated validity. The formulations can then serve both as a simulation tool and an experimental tool for course-of-action development. In the meantime, the main contribution of this formulation is to provide the necessary focus for the KBD and EBA.

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1. INTRODUCTION

NATO, through the Experimentation Program of the Allied Command Transformation (ACT), is conducting experiments with doctrine and tools for Effects-Based Operations (EBO). The ACT development of NATO EBO concepts is closely coordinated with the US Joint Forces Command (JFCOM), which leads the overall multinational EBO concepts development effort. The next major event in a series of exercises is the Multinational Experiment 4 (MNE-4) in February and March 2006. The main NATO C3 Agency (NC3A) contribution to this is the EB-TOPFAS (Effects-Based Tool for Operational Planning, Force Activation and Simulation) now under development by the same team that developed the "Classical" TOPFAS in support of the current NATO Operational Planning Process (OPP) and doctrine. EB-TOPFAS is the planning tool for Effects-Based Planning (EBP). The other main functions of EBO are the Effects-Based Execution (EBE) and the Effects-Based Assessment (EBA). All three functions (Effects-Based Planning, Execution and Assessment) depend on and contribute to the fourth main function, the Knowledge Base Development (KBD). Doctrine and tools for all four functions will be tested in MNE-4. For further documentation of the current state of EBO concepts see Reference 1. For a concise definition of EBO concepts and terminology see Reference 2. Reference 3 includes a more thorough discussion of the basic ideas and rationale for EBO. For a recent study on the particular requirements for situational awareness and understanding in EBO see Reference 4. For current NATO guidance on further development of concepts for the Effects-Based Approach to Operations (EBAO) see Reference 5.

Although it may be said that all good commanders have always conducted effects-based operations it is equally clear that the current development and formalization of EBO concepts extends the traditional framework for the planning and conduct of military operations. This is directly represented by extending the operational space to include all the dimensions of the Political, Military, Economic, Social, Infrastructure and Information (PMESII) environment. The increase in dimensionality necessarily increases the complexity of the planning and execution with the associated need for extended expertise in all PMESII dimensions and analytical support to tie the strands together into a coherent operational design. To simplify the text in the discussion below, the term "military operations" is used to refer to operations that involve all of the PMESII dimensions.

The purpose of this Note is to provide a coherent analytical framework for EBO by mapping the EBO functions and concepts to well known engineering formulations and techniques from the disciplines of state-space analysis and optimal control system design. In the interest of simplicity and to focus on the main conceptual points, the mathematical details and rigor is largely ignored but can be consulted in any standard optimal control system text. See for example References 6, 7 and 8. The relationships between the dimensions of the vectors and matrices in the expressions below are not explicitly noted, but should be obvious from the context. The superscript T (e.g. x^T) is used to denote the transpose of a vector or matrix.

2. OPERATIONAL DYNAMICS AND CONTROL

A key element of the Knowledge Base Development is the System-of-Systems Analysis (SoSA) to identify the operationally relevant elements within the PMESII dimensions and the relationships between the elements, both within and among the PMESII domains. This is sometimes represented by a network of nodes (elements) and links (relationships). See for example Reference 2. The nodes may represent individuals, groups, organizations, forces, assets, installations or any other element that has been identified and defined as relevant for the operation at hand. The system-of-systems in the SoSA is of course a system in its own right and the techniques of systems analysis applies without the need for any special adaptation. However, the separation into political, military, economic, social, infrastructure and information components is helpful in mapping out the operational responsibilities and the needs for domain expertise. Associated with each of the elements (nodes) is a set of attributes (variables) that can take on different values. Although some values might normally be expressed in qualitative terms (good-bad, high-low, etc), in the following it is assumed that all attributes are translated into quantitative (numerical) terms. The elements and the attributes of the PMESII domains collectively form the state vector for the complete operational environment, including own and opposing forces and any other actors or elements that has been identified and selected for explicit representation in the operational planning, execution and assessment. Formally the state vector is defined as the column vector

$$x = [x_i]^T = [x_{Pi}, x_{Mi}, x_{Ei}, x_{Si}, x_{Ii}, x_{Ji}]^T \qquad (2.1)$$

where x_{Pi}, x_{Mi}, x_{Ei}, x_{Si}, x_{Ii}, x_{Ji} are simply the subsets of the overall state vector that quantifies the state of affairs in the political, military, economic, social, infrastructure and information domains. (A particular node and variable may of course be common to two or more PMSEII dimensions.) In EBO terminology the effects are the changes in the state variables x_i in the course of the operation, resulting mainly from the actions undertaken. Needless to say, the state vector may be very large (many elements and attributes) in any real operational context where the descriptions of all relevant elements in all PMESII domains are included. A key task for both the analysts and domain experts will be to extract and synthesize the information in the knowledge base to form a manageable and, at the same time, sufficiently complete representation of the actors and the environment. For the implementation of EBO this will become an essential part of the operational art. In the state-space formalizations below the full scope of the problem is addressed head on; however the EBO ambitions and challenges remain the same whether one adopts this or any other formalism.

If all operational elements and relationships could be mapped out in full by the SoSA, and if the values of the state-vector elements where known at some time $t = t_0$, the future evolution of the state of affairs would be fully and deterministically described by the differential equations:

$$\dot{x} = dx/dt = f\{x(t), t\}; \; x = x(t_0) = x(t) \text{ for } t = t_0 \qquad (2.2)$$

where f{*} is the vector of functions that contain all the SoSA results describing how the values of all elements of the state vector influence the rate of change of the same elements over time. The time $t = t_0$ may represent the start of the operation or more usefully some time prior to the start of the operation when the preparation of the knowledge base is sufficiently mature to support initial SoSA assessments. Equation (2.2) assumes that the state and evolution of the operational environment and actors can be modeled as continuous, deterministic and differentiable processes. This is of course totally unrealistic and in the developments below the necessary adjustments are made to make the representation more useful as an EBO roadmap and possibly suitable for actual computations. For simplicity of notation the continuous representation is retained in the preliminary description of the basic concepts.

The first and immediate extension required to equation (2.2) is the explicit representation of the inherent uncertainty associated with military operations, (or any other operations for that matter). The need for explicit representation of uncertainty in EBO is highlighted in Reference 3, but for some reason is largely ignored in subsequent writings; e.g. References 1-2. Any or all of the state vector elements may be influenced (disturbed) by stochastic processes that represent both the fact that the SoSA is incomplete and inaccurate and that the real evolution of the operation environment will be influenced by unpredictable events and developments. For the time being the relevant stochastic processes w(t) are included by the simple extension to 2.2:

$$\dot{x} = dx/dt = f\{x(t), w(t), t\} \qquad (2.3)$$

Furthermore, full and detailed knowledge of x_0 at $t = t_0$ is no longer assumed. Details are given further on in this report about the “lack of knowledge” and what additions are needed in terms of modeling (assumptions) of the stochastic processes.

Situational awareness, situational understanding and an estimate of the likely future development (without interference) prior to the design of any operation are illustrated in Figure 1. To be able to draw the illustration only two of the multitude of variables is depicted. The variables might for example represent the degree of violence among factions and the terrain occupied by invading forces. The region of initial uncertainty for the two state variables is also included. The interpretation might be analogous to the CEP (circular error probable) concept used in artillery and air-to-ground targeting; i.e. there is a 50% certainty that the values fall within the boundaries.

In addition to the expected development of the state-vector values, Figure 1 also includes the outline of what is considered to be a “region of stability”. If the system somehow could be brought to a state with variable values within this region, the system would be expected to continue “normal” acceptable evolution within the region of stability without deterioration to unacceptable variable values. In terms of the two variables depicted here, the point indicated might therefore be specified as part of the Desired End-State.

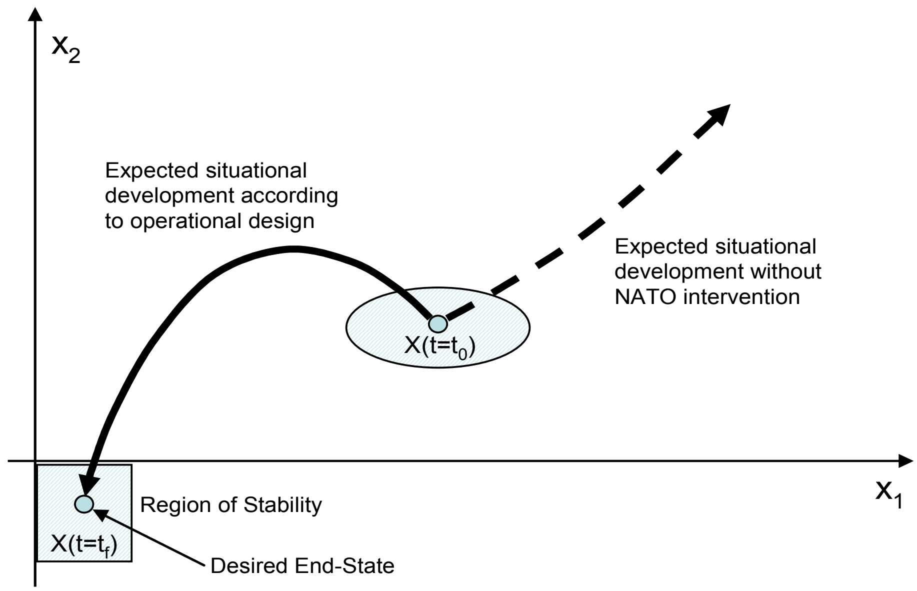

Figure 1 Evolution of operational effects through the state-space

Left to its own devices and random disturbances, Equation 2.3 describes the expected development of the system. However, the background for the operation at hand is that the expected development is considered unacceptable as measured by one or more of the components of the state vector. The purpose of the operation is to create the necessary effects to change the development and to bring the situation to, or as close as practically possible to, the desired end state. The effects are achieved in the course of operation by a series of synchronized actions u(t). In engineering terminology u(t) is the control vector sequence. In EBO terminology the actions belong to one of the four DIME domains (Diplomatic, Informational, Military, Economic); i.e.

$$u = u(t) = [u_a]^T = [u_{Dar}, u_{Iar}, u_{Mar}, u_{Ear}]^T \qquad (2.4)$$

As with the state vector, each of the control vector components (u_{DI}, u_{II}, u_{MI}, u_{EI}) has any number of sub-components that define the potential actions <u>and</u> the candidate resources (means) to achieve the actions. Including the actions in the overall system development changes the complete mathematical equation for the system development to:

$$\dot{x} = dx/dt = f\{x(t), w(t), u(t), t\} \qquad (2.5)$$

where the functions f{*} now include the full descriptions of the rate of change of the elements of the state vector at time t as a result of actions u(t) and the random disturbances w(t) with the system in the state x = x(t). In EBO terms the function f{*} defines the E-N-A-R relationships (Effects-to-Node-to-Action-to-Resource) with the addition of the particular attributes of the nodes that are affected. Recall that both the nodes and their attributes are components of the state vector and that both the potential actions and the associated resources are components of the control vector. At the planning stage the operational design consists of the description of the desired effects over time, x(t) including the Commanders Approved Effects List (CAEL) to be achieved by the sequence of actions and resource assignments, u(t).

The task of the analysts and the domain experts is firstly to define the elements of x(t), u(t) and w(t) and the statistical properties of w(t). Secondly, define the relationships between these in the form of f{*}; and thirdly to estimate the actual state of affairs x(t) at each stage of the operation and to calculate the appropriate sequence of actions u(t) to bring the system from the initial state $x_0 = x(t_0)$ to $x_f = x(t_f)$, the final desired end state at the end of the operation. Needless to say, this general formulation is not very useful as practical guidance for any actual operation of interest and in the developments below the formulation is refined in several respects, but first a few further introductory points.

3. OBSERVATION AND MEASURES OF PERFORMANCE AND EFFECTIVENESS

The state of affairs in the system at any time prior to or during the operation is described by the values of the component variables of the state vector. In the ideal world, to conduct the operation in the best possible manner, these values would be known at all times. However, in any real operation there will not be such perfect knowledge. Many of the key variables (like the capabilities and intentions of individuals or groups) will not be directly observable in real time and will have to be inferred from the observation or measurement of other factors. Furthermore, these observations and measurements are subject to uncertainty and/or random disturbances. The observation/measurement process can be modelled as:

$$z = z(t) = [z_m]^T = h\{x(t), t\} + v(t) \qquad (3.1)$$

where z(t) is the column vector of measurements (quantified observations); x(t) is the state vector; v(t) is the "measurement noise"; i.e. the stochastic sequence of random disturbances to the measurements; and h{*} are the functions that serve as the model of the measurement process. (Note that the dimension of the measurement vector z is not necessarily the same as the dimension of the state vector.) Another key task of the analytical support for the effects-based execution and assessment will be to estimate the state of the system from the measurement process; more on this in Chapter 4.

The specification and monitoring of the Measures of Effectiveness (MOE) and Measures of Performance (MOP) is a major feature of the EBO concept and has been singled out as a separate function, the Effects-Based Assessment (EBA). The MOPs relate to the degree that the planned actions u(t) have been achieved. The MOEs relate to the degree that the desired effects x(t) have been achieved. The ultimate measure of effectiveness is of course the degree to which the operation brings the system to the desired end state. The time available for this may be given as part of the strategic guidance, or the minimization of the time it takes to achieve the end state may be part of the overall objective.

In addition to the overall objective of reaching the desired end state, there will be numerous constraints and restraints imposed as part of the overall political and military strategic guidance. Also, there will be the commanders guidance for the operational design in the form of the Commander's Approved Effects List (CAEL) and the Synchronization Matrix which holds the overall relationships and dependencies between desired intermediate effects, planned actions and the designated resources over the time span of the operation. One of the commander's concerns will be to balance the inevitable trade-offs between the numerous and sometimes conflicting considerations of desired effects and acceptable means. Mathematically, the totality of the operational objectives and guidance can be summarized in one overall performance index:

$$J = \varphi\{x(t_f), t_f\} + \int_{t_0}^{t_f} L\{x(t), u(t), t\}\,dt \qquad (3.2)$$

where φ{*} is the function that represent the degree to which the operation reaches the desired end state. L{*} is the function that represents all the intermediate effects, actions and operational design considerations of the CAEL and the Synchronization Matrix. All the MOP's and MOE's other than the desired end state are incorporated in the function. Referring to Figure 1 it should be clear that in principle there are an unlimited number of paths through the state space from the initial state to the desired end state or acceptable region of stability. The values of the sequence of actions that minimizes (3.2) defines the "optimal" operational design. Depending on the "strength" or weights of the MOEs, MOPs, constraints and restraints included in L{*}, it may or may not be possible to reach the desired end state.

The effects-based operational design task can now be summarized in the terminology of optimal control system design: Determine the control sequence (actions and resources) u(t) that minimizes the overall performance index J based on the model of the overall system dynamics of Equations 2.5 and the model of the measurement process (observations, intelligence and analysis) described by Equations 3.1.

Again, in control system terminology, the situation where the system is in an (unacceptable) initial state and where reaching the end state is the overriding concern is referred to as a "terminal control problem". An engineering analogy might be the missile intercept problem. The analogy to types of potential NATO operations might be initial entry, evacuation or peace enforcement. Time may typically be an important element of such operations. The situation where the system has already been brought to the acceptable region of stability and the task of the control system is to keep it there is referred to as the "regulator problem". An engineering analogy might be an air conditioning system or an offshore drilling rig positioning system. The analogy to types of potential NATO operations might be peacekeeping operations where the situation has already been brought to an acceptable state by diplomacy and where the NATO task is to ensure that the system continues to evolve within acceptable bounds or terms laid down in a treaty or UN resolution. Minimization of the time of the operation will typically not be a major concern.

The general principles of the optimal control system engineering approach to effects-based operations are represented by Equations 2.5, 3.1, the underlying variables, stochastic processes and the minimization of the performance index 3.2. However, the general formulation is much too generic to serve as a guide for the design and execution of effects-based operations and certainly no practical general approach to actually calculating EBO can be based on the general formulations above. Both for these reasons and to better match the formulation to how operations are planned and conducted, two major modifications are introduced.

4. OPTIMAL EBO IN DISCRETE TIME, MULTI-STAGE LINEARIZED FORM

The formulations above represent both the system dynamics and the control sequence (actions) in continuous time in the same way that you would for driving a car or a guided missile intercept. This is not how military operations are planned and conducted. The commander does not have a magical "joy-stick" by which he can steer the operation in continuous real time and space. Rather, the operation is typically planned in several stages, preparation, deployment, etc which are further refined during the execution in decision cycles of 24 hours, 72 hours, or whatever cycle is appropriate for the operation and level of command. This is the familiar OODA loop (Observe – Orient – Decide – Act). It suggests that the mathematical relationships be expressed in discrete (multi-stage) time steps and difference equations rather than continuous time and differential equations. This is particularly true for the control sequence of actions that need to be tied to the decision cycle. Some of the underlying dynamics of the operational environment may best be represented as continuous time processes, but these relationships can easily be translated into equivalent multi-stage representations by standard techniques. The discrete multi-stage versions of 2.5 and 3.1 are simply:

$$x_{k+1} = f_{k+1|k}(x_k, w_k, u_k) \qquad (4.1)$$

$$z_{k+1} = h_{k+1}(x_{k+1}) + v_{k+1} \qquad (4.2)$$

where k = 0, 1, 2, 3, ... , T. The final time, T may or may not be specified. The subscript notation k+1|k signifies that the function describes the state transition from time k to k+1 as functions of the state (x_k), control/action (u_k) and the disturbance (w_k) at time k. (For simplicity the values of the state vector are measured relative to the desired end state; i.e. $x_T = 0$. Also note that the subscripts, k, k+1, etc in the following denotes time or stage rather than vector component.) For decision superiority in a network centric setting the OODA cycle ($\Delta t = t_k - t_{k-1}$) should be kept as short as possible.

The second major modification is to base the remainder of the discussion on a linearized representation of the system dynamics and the measurement process. In general, the functions in (2.5) and (3.1) will of course not be linear but once the desired path has been mapped out through the state space, as defined by the CAEL and the operational design, a practical approach may be to linearize the relationships around the planned course of action. The initial development (calculation) of the operational design, recommended CAEL and E-N-A-R at the planning stage must of course be based on the full and typically non-linear relationships. (Further points on this in Chapter 5.) However, for the present purposes the linearized relationships are better suited to highlight the basic concepts and the suggested program for analytical support of EBO in the execution and assessment.

It should also be noted that the system identification, defining $f_{k+1|k}(*)$ and $h_{k+1}(*)$, is the point where the analogy between control engineering and military operations is the weakest. Standard control system design is typically based on well-understood laws of nature or engineering approximations. For the complex interactions in the PMESII dimensions among the actors and other nodes in a military operation there is no comparable scientific basis. The relationships in the equations will therefore be postulations based on assumptions and hypotheses that cannot be tested to a degree of confidence similar to that of the engineering disciplines. This does not mean that the search should be abandoned for quantifiable basic relationships of common validity for the types of operations of interest. As EBO expertise grows it must be expected that lessons of value to future operations can be refined to expressions of quantifiable relationships and parameter values. In any particular operation the functional relationships and parameter values must, of course, be subject to continued assessment and revision as part of the EBA.

There are two further challenges to the system identification for military operations. Firstly, each operation is in many respects unique and there is no option for extensive experimentation with operational concepts in a controlled environment. In missile engineering, for example, there will be numerous test launches to validate the engineering assumptions. Military operations on the other hand are conducted once and the operational environment and objectives for the next one will always differ to a greater or lesser extent. Validation of underlying quantitative relationships of general validity will therefore be tentative at best. Secondly, the military operations are typically conducted against, or in the presence of, adversaries who are free (to a greater or lesser extent) to adapt their actions in response to ours. In the Cold War era this was a dominant consideration, which complicated the analytical support for the development of strategic options. The principles of game theory represented that strategic environment better than control theory. (Control theory can be extended to include differential games, but only with associated computational complexity.) Adversarial actions developed in response to ours will always be an aspect of military operations, but an implicit assumption in the present NATO security environment is that the type of operations that NATO is likely to become involved in are those in which the combined power of the member nations and partners provides an operational setting that allows operation according to the principles of control rather than all-out war between equally powerful adversaries. Asymmetries between the actors in terms of objectives and acceptable means to achieve them only serve to emphasize this point.

In summary, the best estimates of the full non-linear forms of the relevant EBO functional relationships are required for the analytical support. The linearized version of these becomes part of the perturbation calculations and also serves to present the basic principles of the state-space approach. The linearized form of equations 4.1 and 4.2 are

$$x_{k+1} = F_{k+1|k}x_k + G_{k+1|k}w_k + C_{k+1|k}u_k \qquad (4.3)$$

$$z_{k+1} = H_{k+1}x_{k+1} + v_{k+1} \qquad (4.4)$$

where $F_{k+1|k}$, $G_{k+1|k}$, $C_{k+1|k}$ are the respective transition matrices calculated as partial derivatives of $f_{k+1|k}(*)$ evaluated at times k = 0, 1, 2, . . for the nominal values of x_k and u_k. In the same manner H_{k+1} is the linearized matrix of the postulated relationships $h_{k+1}(*)$ between the actual state variable values and the measurements/observations/intelligence. Also note that the variables x_k, u_k, etc in (4.3) and (4.4) are deviations from the nominal values in (4.1) and (4.2), and should correctly be represented by the notation Δx_k, Δu_k, etc. For simplicity the Δ is dropped from the subsequent expressions, but in any implementation of this approach it must be remembered that the full values of the state and control variables are calculated as the sum of the values from (4.1) and (4.3). Furthermore, the linearizations of $F_{k+1|k}$, $G_{k+1|k}$, $C_{k+1|k}$ and H_{k+1} should be re-evaluated at the sequentially improved estimates of the state and control variables as the operation progresses.

The explicit representation of the operational uncertainties in (4.1) - (4.4) in the form of the random process (disturbances) w_k and the measurement uncertainty v_k means that also x_k and z_k are random processes. Without other data for the statistical properties of the processes, a common approach in control engineering is to model the disturbances and measurement uncertainties as Gauss-Markov zero mean processes. This means that the underlying statistics are those of the joint Normal distributions, which are completely described by the zero mean and the covariance matrices, Q_k and R_k for the system disturbances and the measurement uncertainties respectively. It also means that any bias (non-zero expectation/mean) in w_k and v_k would be represented in $F_{k+1|k}$ and H_k. Furthermore, it means that the disturbance (and measurement uncertainty) at one stage is independent of earlier disturbances. Cases where the disturbance at one stage is related to disturbances at earlier stages are modeled by extending the state vector and incorporating the relationships in the system model $f_{k+1|k}\{*\}$. Since x_k is a collection of random processes observed only indirectly through the measurement process (4.4), the first step towards calculating the optimal sequence of actions (responses) is to estimate the state, $\hat{x}_k$ from the observations z_k. The optimal estimates that minimize the sum of the expected mean square errors of the estimate are calculated from the Kalman filter equations:

$$\hat{x}_{k+1} = F_{k+1|k}\hat{x}_k + C_{k+1|k}u_k + K_{k+1}[z_{k+1} - H_{k+1}F_{k+1|k}\hat{x}_k]; \quad \hat{x}_0 = 0 \qquad (4.5)$$

where

$$K_{k+1} = P_{k+1|k}H^T_{k+1}[H_{k+1}P_{k+1|k}H^T_{k+1} + R_{k+1}]^{-1} \qquad (4.6)$$

$$P_{k+1|k} = F_{k+1|k}P_{k|k}F^T_{k+1|k} + G_{k+1|k}Q_kG^T_{k+1|k} \qquad (4.7)$$

$$P_{k+1|k+1} = [I - K_{k+1}H_{k+1}]P_{k+1|k} \qquad (4.8)$$

$P_{0|0}$, the initial value for $P_{k|k}$ in Equation (4.7), is the estimate of the uncertainty (covariance) of knowledge of the initial state x_0, at the start of the operation.

The interpretation of the covariance calculations is that $P_{k+1|k}$ is the covariance of the estimate $\hat{x}_{k+1}$ prior to the z_{k+1} measurement and $P_{k+1|k+1}$ is the covariance after the measurement. The explicit representation and handling of the operational uncertainties is one of the main contributions of the state-space approach to EBO.

The further (re)-calculation of the optimal action-resource sequence u_k requires definition of an optimization criterion analogous to Equation (3.2). As discussed in Chapter 3, the optimization criterion represents the combination of the desired end state and all other considerations of intermediate effects and actions from the start of the operation (k = 0) to the end (k = T). There is no unique way to derive the right form for the optimization criterion. However, in control engineering the expected value of the following quadratic form has been found useful, both in terms of facilitating the calculations and by representing the relevant trade-offs between the end result x_T, the intermediate effects x_k and the action-resource sequence u_k.

$$J = E\{ x^T_T A_T x_T + \sum_{k=1}^{T-1} [x^T_k A_k x_k + 2x^T_k N_k u_k + u^T_{k-1} B_{k-1} u_{k-1}]\} \qquad (4.9)$$

where the values of the A, B and N matrices are selected by judgment to represent the relative weights/importance between the end state (A_T), the intermediate effects (A_k), the actions (B_k), and any combinations of effects and actions (N_k) that may be included. For initial developments it is suggested that the weighting matrices be kept as simple as possible; e.g. A and B diagonal and N = 0. With the performance index of Equation (4.9), the optimal control sequence is calculated from the equations:

$$u_k = S_k \hat{x}_k \tag{4.10}$$

where S_k is calculated recursively backwards from k = T from the equations:

$$S_k = - [C^T{}_{k+1|k} W_{k+1}\, C_{k+1|k} + B_k]^{-1} [C^T{}_{k+1} W_{k+1} F_{k+1|k} + N^T{}_k] \tag{4.11}$$

$$W_k = F^T{}_{k+1|k} W_{k+1}\, F_{k+1|k} + F^T{}_{k+1|k} W_{k+1} C_{k+1|k\ k} + A_k \tag{4.12}$$

for k = T-1, T-2, . . . , 0 with A_T as the initial value for W_T in Equation (4.12).

The full derivation of these equations with the associated necessary conditions on the parameters and relationships can be found in References 6, 7 and 8. Equations (4.11) and (4.12) may be interpreted to represent a structured method for bringing the objective of the end state recursively backwards through to the earlier stages of the operation. Equation (4.10) represents the so-called “certainty equivalence principle” that allows use of the state estimate $\hat{x}_k$ to calculate the optimal control sequence u_k in the same way as would be done in a deterministic environment without stochastic system disturbances or measurement errors.

In order to maximize the use of the evolving knowledge base, the calculations and evaluation/updating of the Equations (4.1) to (4.12) should be repeated in each decision cycle. As the operation progresses the current estimates of the situation becomes the starting point for the remainder of the operation. If the action-resource sequence in each decision cycle is reviewed/revised and re-calculated, the only calculated result actually used each time is u_0.

Needless to say, the dimensions of the vectors and matrices in the above expressions will be very large for even a high-level representation of a real operation. Both the establishment of the functional relationships and the actual calculations may be beyond near-term expertise and capabilities. However, as a minimum it is suggested that the basic concepts of the state-space approach be implemented to provide the necessary focus for the knowledge base development and the assessment processes.

5. SUMMARY DISCUSSION

The implementation of the state-space approach for the derivation of optimal action-resource sequences to reach the desired end state with due consideration of the CAEL and other constraints or restraints can be pictured as four main processes tied together in a feedback loop as illustrated in Figure 2.

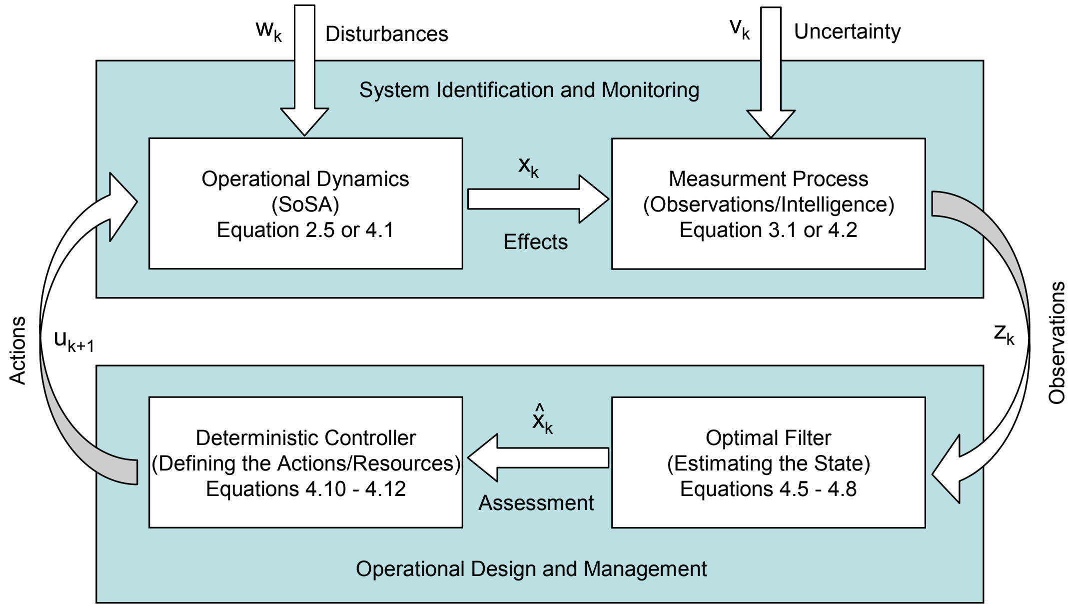

Figure 2 Schematic of the EBO state-space formulation relationships

The analytical support process can be summarized in the sequence of steps described below.

5.1 SYSTEM IDENTIFICATION

This is the main purpose of the knowledge base development, leading to the formulation of the functional relationships in Equations (4.1) through (4.4). This includes the determination of the relevant PMESII effects and node attributes to include in the state vector x_k; the potential factors to include in the disturbance vector w_k; the DIME actions and appropriate resources to include in the action-resource vector u_k; as well as the elements of the observation-intelligence vector z_k. This is, of course, a most challenging task and will require the combined talents of military experts and operational analysts. Although challenging, the task is similar to that faced by the developers of simulation models. Reference 9 describes the GAMMA model that has been designed to simulate traditional as well as asymmetric operations of direct relevance to EBO. The agent based model design principles may serve as a starting point for the system identification.

5.2 ESTIMATION OF OPERATIONAL PARAMETERS

Once the initial formulations of the system and measurement models are in place, estimate the parameter values and initial conditions for Equations (4.1) – (4.4). This includes the estimate of the initial value for the state vector x_0 and degree of uncertainty associated with this ($P_{0|0}$); as well as the statistical properties of the disturbance (Q_k) and the measurement uncertainties (R_k).

5.3 OPERATIONAL DESIGN

At the advance planning stage the key task is to develop the concept of operation in terms of the *a priori* planned values for the action-resource sequence u_k. Referring to

Figure 1, this becomes the plan to bring the system along the nominal path through the state-space from the initial state x_0 to the desired end state x_T. This amounts to solving the deterministic optimal control problem described by Equation (4.1) without the unknown disturbances w_k; i.e. $x_{k+1} = f_{k+1|k}(x_k, u_k)$. In the unlikely case that the full equations are linear, or can adequately be approximated by linear equations ($x_{k+1} = F_{k+1|k}x_k + C_{k+1|k}u_k$), equations (4.10) – (4.12) provide the solution. (In this case it would be advisable to seek to transfer the complete formulation to a Linear Programming formulation. This would dramatically simplify the calculations and open for further explicit representation of operational constraints and restraints.) In the more likely case where the functions in Equation (4.1) include significant non-linearities, direct numerical calculation of the operational design will be difficult, to say the least. However, there is always the “manual” operational planning as a potential starting basis. After translating this into numerical terms as an initial estimate for u_k, the deterministic version of (4.1) can be solved. It is unlikely that the numerical end result matches the desired end state, but it may serve as the starting point for sequential refinement of the operational design. Chapter 7.7 of Reference 6 suggests a backward sweep algorithm for iterative improvements of u_k. The solution would be the analytical contribution to recommendations for the CAEL and the synchronized E-N-A-R sequence. The sequences u_k and x_k are also the nominal values and basis for the linearizations (4.3) and (4.4).

5.4 EXECUTION AND ASSESSMENT

No plan survives the contact with reality. The same will be true for the nominal *a priori* operational design. However, having laid the solid groundwork in the system identification, parameter estimation and the operational design at the planning stage, the analytical support in the EBO execution and assessment can concentrate on the implementation of the processes defined by Equations (4.5) – (4.12); i.e. estimate the current state of the operation relative to the desired end state and the nominal values at the current stage (decision cycle) according to the operational design; followed by development (calculation) of the recommended action-resource response; and the review/revision as required of the analytical formulations.

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ABBREVIATIONS

ACT	Allied Command Transformation
CAEL	Commanders Approved Effects List
CEP	Circular Error Probable
DIME	Diplomatic, Informational, Military, Economic
EBA	Effects-Based Assessment
EBAO	Effects-Based Approach to Operations
EBE	Effects-Based Execution
EBO	Effects-Based Operations
EBP	Effects-Based Planning
EB-TOPFAS	Effects-Based Tool for Operational Planning, Force Activation and Simulation
E-N-A-R	Effects to Node to Action to Resource
JFCOM	Joint Forces Command
KBD	Knowledge Base Development
MNE-4	Multinational Experiment 4
MOE	Measures of Effectiveness
MOP	Measures of Performance
NC3A	NATO C3 Agency
OODA	Observe, Orient, Decide, Act
OPP	Operational Planning Process
PMESII	Political, Military, Economic, Social, Infrastructure and Information
SoSA	System-of-Systems Analysis
US	United States