A Sufficient Comparison of Trackers

David Bizup
Systems and Information Engineering
University of Virginia
dfb2ph@virginia.edu

Donald Bizup
Systems and Information Engineering
University of Virginia
brown@virginia.edu

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Need for Sufficient Comparisons

- Peculiar deficiency in the literature, trackers judged on
  - one or two measures of performance
  - one or two trajectories
- Fair method, based on sufficient comparisons:
  - Define comprehensive set of maneuvers
  - Define relevant measures of performance
  - Generate simulated radar data for every trajectory
  - Run data through each tracker
  - Compare error distributions via sufficiency

Trajectories and Measures

- 15,120 constant rate turns:

Trajectory Parameter	Levels
Range at start of maneuver	20, 40, ..., 300 miles
Initial heading	0, 30, ..., 330 degrees
Speed	200, 300, ..., 800 knots
Centripetal Acceleration	0, 1, ..., 5 g
Turn Direction	left, right

- Measures of Performance
  - Position accuracy
  - Speed accuracy
  - Heading accuracy
  - Range accuracy
  - Bearing accuracy
  - Range rate accuracy
  - Cross-range rate accuracy

Trackers

- Two mode interacting multiple model
  - 1 nonmaneuvering mode
  - 1 maneuvering mode
  - Standard for tracking maneuvering tarets
  - Bar-Shalom and Li, *Multitarget-Multitarget Tracking: Principles and Techniques*, YBS (1995)
- Single mode, adaptive c_{min} Kalman filter
  - Use range rate to produce statistic of accelerations (7^{th} ICCRTS)
  - Gain set based on the statistic
  - Simpler than IMM
  - Uses additional information

Statistical Sufficiency

- Probabilistic sufficiency is analagous to more familiar statistical sufficiency
  - If *A* is sufficient for θ, then B contains no more information about θ than *A*
  - *B* can be interpreted as *A*, plus noise
- Analogy is imperfect
  - *A* is either a sufficient statistic, or not
  - Concept of minimally sufficient statistic
  - Sufficiency of *A* does not depend on other statistics

Probabilistic Sufficiency

Blackwell, *Equivalent Comparisons of Experiments*,
Annals of Mathematical Statistics, vol. 24, pp265-272 (1953)

- Relationship between estimators
  - Either ASB, or BSA, or neither, or both
  - Possible that CSA and CSB
  - Establishes a partial order
- Definition: ASB if B is a stochastic transformation of A
  - B can be interpreted as A, plus noise
- Powerful, because if ASB, then A is better than B
  - A has a lower Bayes risk for <u>any</u> prior, <u>any</u> loss function

Informativeness

[**Definition: Informativeness**] It is said that estimator A is more *informative* than estimator B, denoted AIB, if $R_A \leq R_B$ for all prior distributions and all loss functions, where R_i is the Bayes Risk of estimator i.

If A is more informative than B, then A is better than B no matter how better is defined!

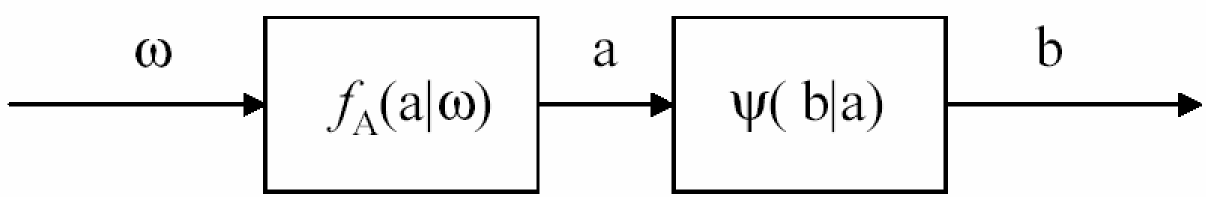
Unfortunately, no constructive way to determine informativeness.

Sufficiency

[**Definition: Sufficiency**] It is said that estimator A is *sufficient* for estimator B, denoted ASB, if there exists a stochastic transformation $\psi : B \times A \to \mathrm{Re}^{+}$ such that the following are satisfied where ω denotes the predictand or true state of nature.

$$
\begin{aligned}
f_B(b|\omega) &= \int_A \psi(b|a)\, f_A(a|\omega)\, da,\ \omega \in \Omega,\ b \in \mathcal{B} \\
\psi(b|a) &\geq 0 \\
\int_B \psi(b|a)\, db &= 1
\end{aligned}
$$

If *ASB*, then we can interpret *B* as *A* plus noise:



Sufficiency => Informativeness

[**Theorem: Sufficiency $\Longrightarrow$ Informativeness**] If estimator A is sufficient for B, then A is more informative B.

- Prove sufficiency parametrically for:
  - Univariate and bivariate Gaussian likelihoods
  - Lognormal likelihoods
  - Beta likelihoods
- Prove by showing existence of certain matrices
  - Multivariate Gaussian likelihoods

Using Sufficiency

- Run the simulations
- We found that, for any measure of performance, the error distributions are approximately Gaussian with scale parameter 1
- Determine sufficiency relationships for each measure of performance, each trajectory
- Condition on relevant trajectory parameters
- Determine proportion of trajectories where *ASB*
- *ASB* does not tell you how much better *A* is than *B*, so we also report variance ratios

Sufficient Comparisons

- Unconditional sufficiency of an adaptive c_{min} tracker for a two mode interacting multiple model, constant rate turns

proportion for which c_{min} *S* imm

Time	2D Pos	Speed	Heading	Range	Bearing	Rng Rt	X-Rng Rt
0 sec	0.9983	0.9987	0.8674	0.9991	0.9993	0.9980	0.9998
10	0.9970	0.9983	0.8585	0.9987	0.9991	0.9987	1.0000
20	0.9870	0.9813	0.8453	0.7776	0.9957	0.7346	1.0000
30	0.9716	0.9039	0.8149	0.6749	0.9816	0.5371	0.9729
40	0.9119	0.7595	0.6873	0.8312	0.9373	0.6886	0.8088
50	0.8103	0.6226	0.6782	0.9486	0.8451	0.9236	0.5816

variance ratios $\sigma^2_{imm} : \sigma^2_{c_{min}}$

Time	2D Pos	Speed	Heading	Range	Bearing	Rng Rt	X-Rng Rt
0 sec	2.9614	4.4665	1.3772	5.5660	2.7444	5.9335	5.8156
10	2.8199	4.4796	1.3943	5.1887	2.6207	5.4865	5.6934
20	2.6004	3.2424	1.4630	1.6261	2.5770	1.4042	5.9144
30	2.4479	2.0881	1.4748	1.2522	2.5622	0.9093	4.3019
40	2.0482	1.5423	1.2602	1.6377	1.7810	1.1234	1.9695
50	1.6060	1.2089	1.1962	2.2325	1.3597	1.8474	1.2156

Summary

- New technique for comparing trackers fairly
- Comprehensive trajectory sets
- Comprehensive measures of performance
- Sufficient comparisons
  - proportions
  - variance ratios
- Future tracker evaluations